

Unit 3: Triangle Congruence

Unit 3 Main Objectives:

- I can compare congruence to rigid motions.
- I can demonstrate that two figures are congruent by using one or more rigid motions to map one onto the other.
- I can use triangle congruence to solve problems.
- I can apply properties and theorems about isosceles and equilateral triangles to solve problems.
- I can prove triangles are congruent by identifying corresponding parts and applying correct congruence theorems.
- I can write triangle congruence statements.
- I can justify statements of a proof with definitions, theorems, and properties.
- I can explain that corresponding parts of congruent triangles are congruent.
- I can apply congruence criteria to intricate problems involving overlapping triangles and multiple triangles.

Key Vocabulary: I can define the following vocabulary terms.

- | | | |
|--|---------------------------------------|---|
| <input type="checkbox"/> Congruent | <input type="checkbox"/> Rigid Motion | <input type="checkbox"/> Corresponding |
| <input type="checkbox"/> ASA | <input type="checkbox"/> SAS | <input type="checkbox"/> AAS |
| <input type="checkbox"/> HL | <input type="checkbox"/> SSS | <input type="checkbox"/> Hypotenuse |
| <input type="checkbox"/> Bisector | <input type="checkbox"/> Leg | <input type="checkbox"/> CPCTC |
| <input type="checkbox"/> Isosceles | <input type="checkbox"/> Equilateral | <input type="checkbox"/> Criteria |
| <input type="checkbox"/> Postulate | <input type="checkbox"/> Theorem | <input type="checkbox"/> Reflexive |
| <input type="checkbox"/> Vertical Angles | <input type="checkbox"/> Midpoint | <input type="checkbox"/> Alternate Interior |

Day 1 - CONGRUENCE STATEMENTS

Explore and Reason

Some corporate logos are distinctive because they make use of repeated shapes.



Part A: Bolt company wants to update their logo. Draw a new logo next to the original.

Original Image



New Logo



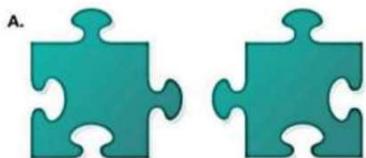
Part B: The owner of the company says “I like your designs, but it is important that the logo be the same size and same shape as the original image.” What would you do to comply with the owner’s requirement?

Part C: Is comparing the measures of all the angles in the preimage and in the image sufficient to guarantee the figures are congruent?

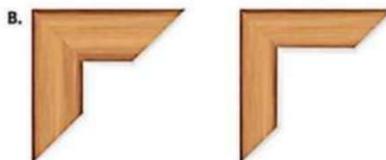
EXAMPLE 4 Determine Congruence

Which pairs of objects are congruent? If a pair of objects is congruent, describe a composition of rigid motions that maps one to the other.

(Complete Try It 4 on Pg. 250)



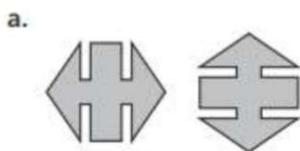
congruent,
reflection



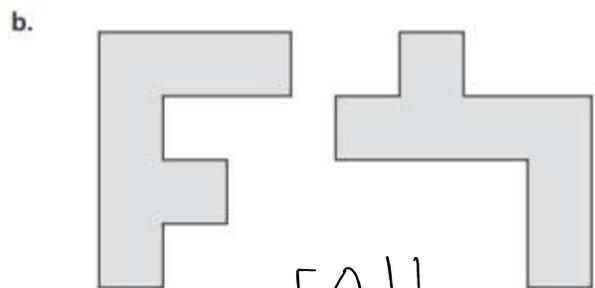
NO!



congruent,
reflect and rotate



congruent,
rotation

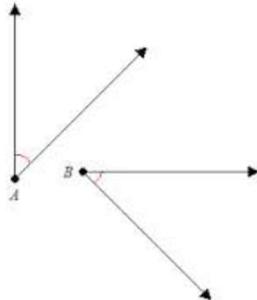


NO!!

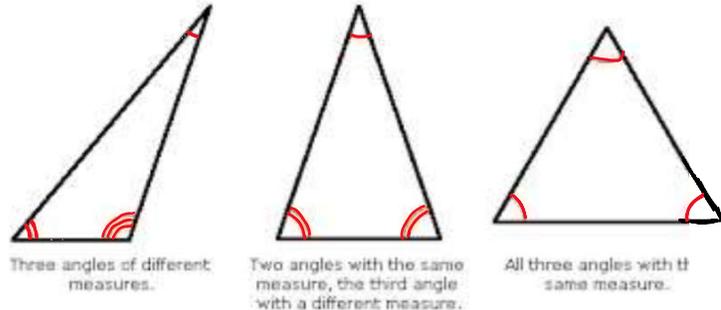
Congruence: Guided Notes

Congruent Angles

→ Angles can be marked with angle markings to determine congruent and non-congruent angles.
Ex.)

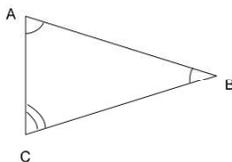


Ex.)

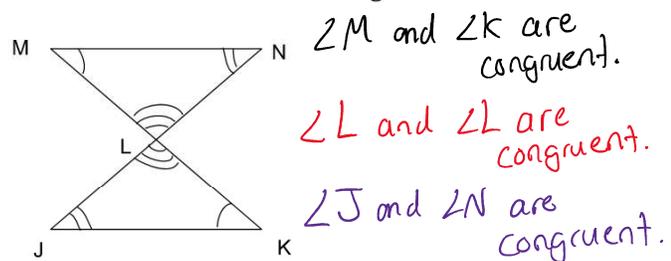


You Try: Which angles are congruent to each other based off of the markings?

$\angle A$ and $\angle B$ are congruent.



You Try: Which angles are congruent to each other based off of the markings?



Congruent Figures (The symbol for "congruent" is \cong)

→ Figures can be stated as congruent using a congruency statement, or a marked picture.

→ If two figures are congruent, then all corresponding sides and angles are also congruent.

→ When naming sides and angles, ORDER MATTERS! Corresponding congruent angles and sides are always named in the same order.

Ex.) Given $\triangle ABC \cong \triangle WXY$

a. $\angle B \cong \angle X$

e. $\overline{AB} \cong \overline{WX}$

b. $\angle Y \cong \angle C$

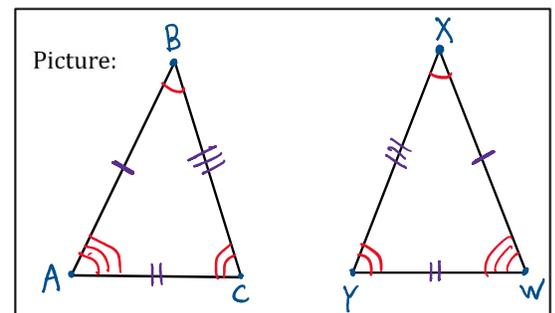
f. $\overline{WY} \cong \overline{AC}$

c. $\angle BCA \cong \angle XYW$

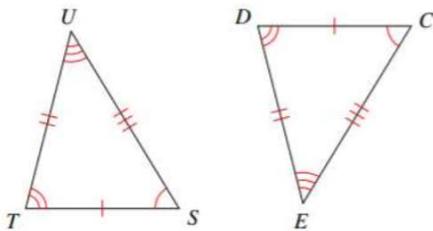
g. $\overline{BC} \cong \overline{XY}$

d. $\angle YXW \cong \angle CBA$

h. $\overline{YX} \cong \overline{CB}$



You Try: Complete all congruency statements based on the diagram below:



a. $\triangle STU \cong \triangle CDE$

b. $\angle U \cong \angle E$

f. $\overline{UT} \cong \overline{ED}$

c. $\angle S \cong \angle C$

g. $\overline{EC} \cong \overline{US}$

d. $\angle T \cong \angle D$

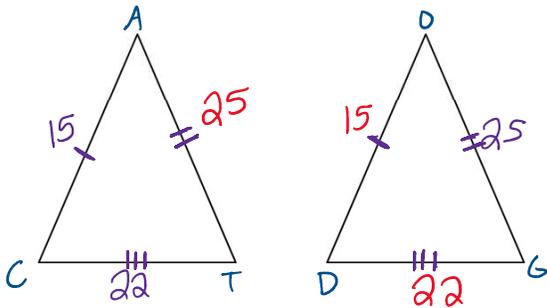
h. $\overline{DE} \cong \overline{TU}$

e. $\angle TUS \cong \angle DEC$

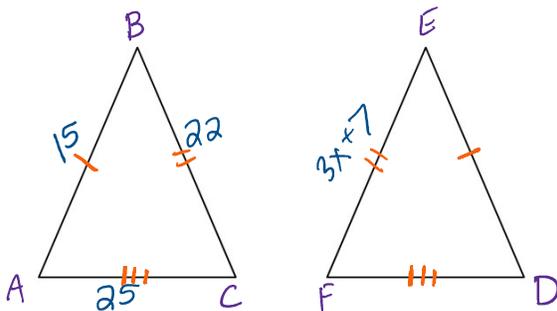
i. $\overline{ST} \cong \overline{CD}$

Finding Missing Sides/ Angles of Congruent Figures

Ex.) Given $\triangle CAT \cong \triangle DOG$, $\overline{AC} = 15$, $\overline{CT} = 22$, and $\overline{OG} = 25$. Using this information, label both triangles and then find the lengths of \overline{AT} , \overline{DG} , and \overline{OD} .



Ex.) Given $\triangle ABC \cong \triangle DEF$, $\overline{AB} = 15$, $\overline{BC} = 22$, $\overline{AC} = 25$, $\overline{EF} = 3x + 7$. Label the triangles and set up an equation to solve for the value of x and \overline{EF}



$$\overline{EF} = 22$$

$$3x + 7 = 22$$

$$\begin{array}{r} -7 \\ -7 \end{array}$$

$$\frac{3x}{3} = \frac{15}{3}$$

$$x = 5$$

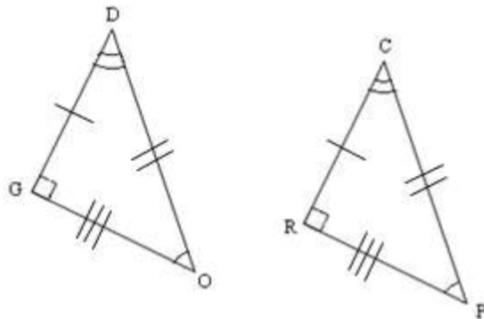
You Try! Classwork

1. Given that $\triangle ARN \cong \triangle JKL$, complete the congruence statements

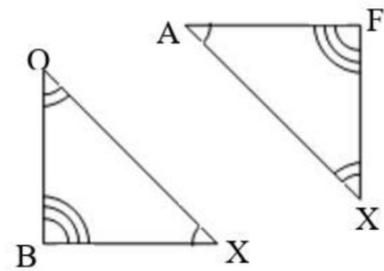
- a. $\angle L = \underline{\angle N}$ b. $\angle A = \underline{\angle J}$ c. $\overline{AN} = \underline{\overline{JL}}$ d. $\overline{JK} = \underline{\overline{AR}}$

Name the following triangles in corresponding order.

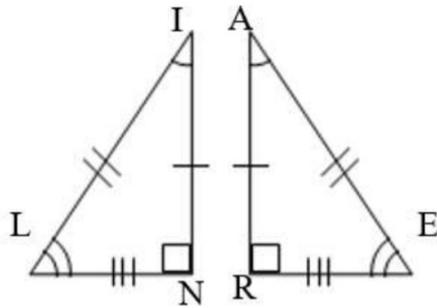
2. $\triangle ODG \cong \underline{\triangle PCR}$



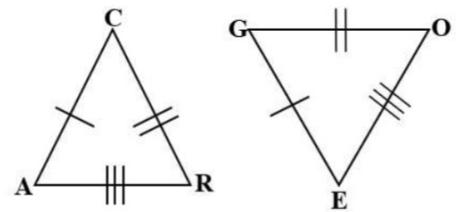
3. $\triangle BOX \cong \underline{\triangle FXA}$



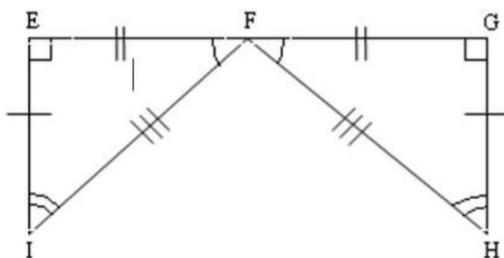
4. $\triangle LIN \cong \underline{\triangle EAR}$



5. $\triangle CAR \cong \underline{\triangle GEO}$

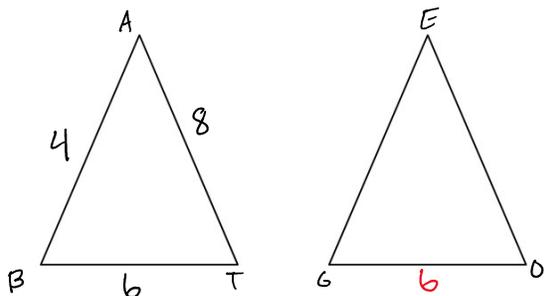


6. $\triangle FEI \cong \underline{\triangle FGH}$



- a. $\angle FEI \cong \underline{\angle GFH}$ d. $\overline{FG} \cong \underline{\overline{FE}}$
 b. $\angle G \cong \underline{\angle E}$ e. $\overline{GH} \cong \underline{\overline{EI}}$
 c. $\angle H \cong \underline{\angle I}$ f. $\overline{FH} \cong \underline{\overline{FI}}$

7. $\triangle BAT \cong \triangle GEO$, $\overline{AB} = 4$, $\overline{BT} = 6$, and $\overline{AT} = 8$. What is the length of \overline{GO} ? How do you know?

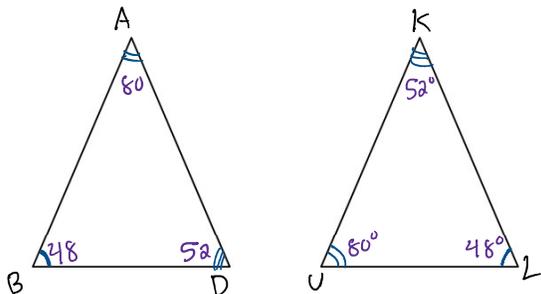


$\overline{GO} \cong \overline{BT}$
 $\overline{BT} = 6$ so $\overline{GO} = 6$

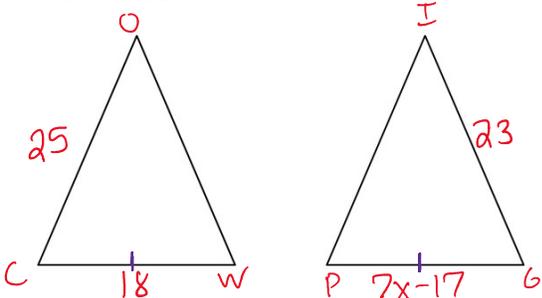
8. $\triangle BAD \cong \triangle LUK$, $m\angle D = 52^\circ$, $m\angle B = 48^\circ$, and $m\angle A = 80^\circ$

a. What is the largest angle of $\triangle LUK$? $\angle U$

b. What is the smallest angle of $\triangle LUK$? $\angle L$



9. If $\triangle COW \cong \triangle PIG$, and $\overline{CO} = 25$, $\overline{CW} = 18$, $\overline{IG} = 23$, and $\overline{PG} = 7x - 17$. Find the value of x and \overline{PG} .



$x = 5$

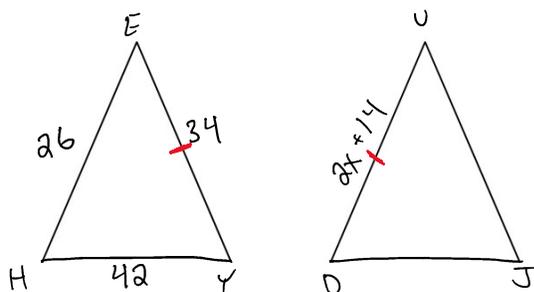
$PG = 18$

$7x - 17 = 18$

$7x = 35$

$x = 5$

10. Given $\triangle HEY \cong \triangle JUD$, $\overline{HE} = 26$, $\overline{HY} = 42$, $\overline{EY} = 34$, and $\overline{DU} = 2x + 14$, find x .



$x = 10$

$DU = 34$

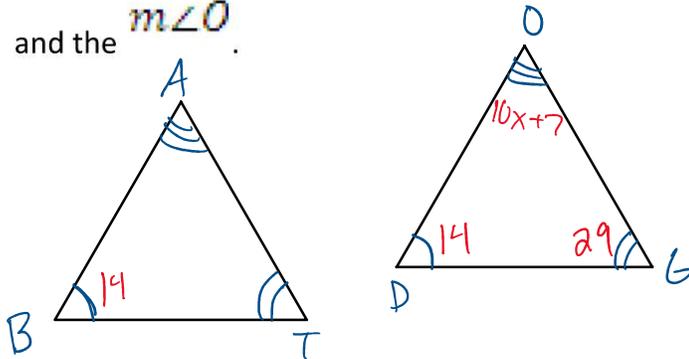
$2x + 14 = 34$

$2x = 20$

$x = 10$

Draw and label a diagram. Then solve for the variable and the missing measure or length.

1. If $\triangle BAT \cong \triangle DOG$, and $m\angle B = 14$, $m\angle G = 29$, and $m\angle O = 10x + 7$. Find the value of x and the $m\angle O$.



$$14 + 29 + 10x + 7 = 180$$

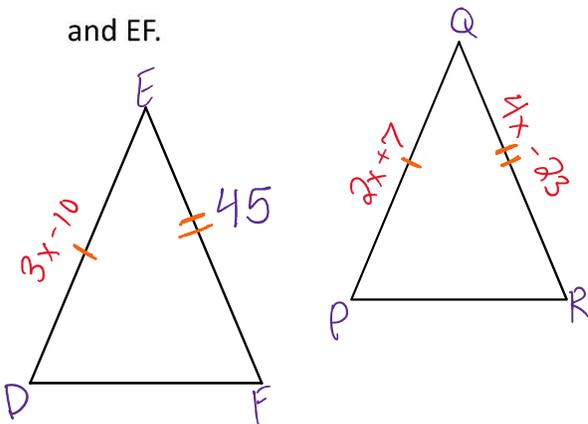
$$10x + 50 = 180$$

$$10x = 130$$

$$x = 13$$

$$x = \underline{13} \quad m\angle O = \underline{137^\circ}$$

2. If $\triangle DEF \cong \triangle PQR$ and $DE = 3x - 10$, $QR = 4x - 23$, and $PQ = 2x + 7$. Find the value of x and EF .



$$3x - 10 = 2x + 7$$

$$x - 10 = 7$$

$$\boxed{x = 17}$$

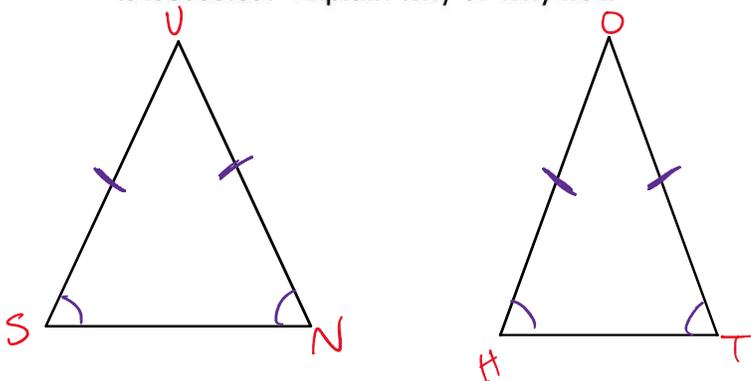
$$4(17) - 23$$

$$68 - 23$$

$$45$$

$$x = \underline{17} \quad EF = \underline{45}$$

3. $\triangle SUN \cong \triangle HOT$. $\triangle SUN$ is isosceles. Is there enough information to determine if $\triangle HOT$ is isosceles? Explain why or why not.



$$\overline{SU} \cong \overline{HO}$$

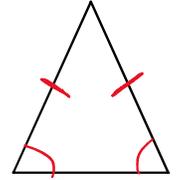
$$\overline{UN} \cong \overline{OT}$$

$$\angle S \cong \angle H$$

$$\angle N \cong \angle T$$

yes, b/c of congruency statement.

Day 2 - ISOSCELES TRIANGLE THEOREM



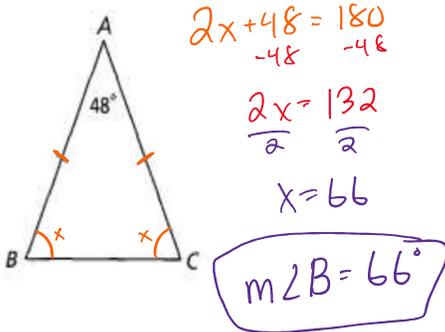
Isosceles Triangle Theorem:

- Isosceles triangles have two congruent sides and two congruent angles.

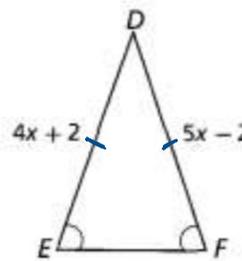
- Isosceles Triangle Theorem:** If two sides of a triangle are congruent, then the two opposite angles will be congruent.

- It works the other way too! If two angles in a triangle are congruent, then the two opposite sides will be congruent.

1. Find $m\angle B$



2. Find \overline{DF}



$$4x + 2 = 5x - 2$$

$$-4x + 2 \quad -4x + 2$$

$$4 = x$$

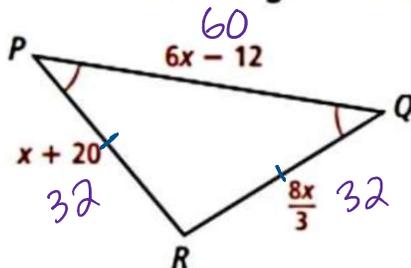
$$\overline{DF} = 5(4) - 2$$

$$= 20 - 2$$

$$\overline{DF} = 18$$

EXAMPLE 3 Use the Converse of the Isosceles Triangle Theorem

What are the lengths of all three sides of the triangle?



$$3(x + 20) = \frac{8x}{3} (3)$$

$$3x + 60 = 8x$$

$$-3x \quad -3x$$

$$60 = 5x$$

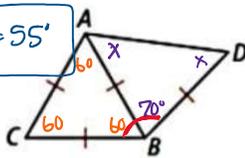
$$12 = x$$

- Similar rules apply for equilateral triangle but now all sides are equal and all angles are equal with a measure of 60 degrees.

EXAMPLE 6 Find Angle Measures in Isosceles and Equilateral Triangles

A. If $m\angle CBD = 130$, what is $m\angle BAD$?

$m\angle BAD = 55^\circ$

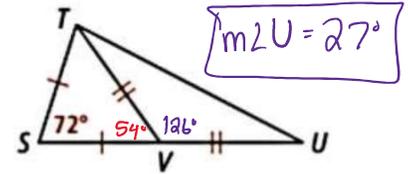


$$\begin{aligned} 2x + 70 &= 180 \\ 2x &= 110 \\ x &= 55 \end{aligned}$$

$$\begin{aligned} m\angle CBA + m\angle ABD &= m\angle CBD \\ 60 + m\angle ABD &= 130 \\ m\angle ABD &= 70^\circ \end{aligned}$$

B. What is $m\angle U$?

$$\begin{aligned} 2x + 72 &= 180 \\ 2x &= 108 \\ x &= 54 \end{aligned}$$

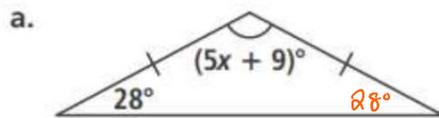


$m\angle U = 27^\circ$

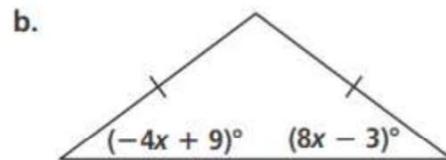
$$\begin{aligned} 2y + 126 &= 180 \\ 2y &= 54 \\ y &= 27 \end{aligned}$$

Try It! Use the Isosceles Triangle Theorem

2. What is the value of x ?

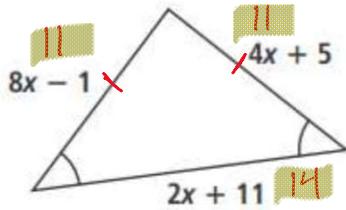


$$\begin{aligned} 28 + 28 + 5x + 9 &= 180 \\ 65 + 5x &= 180 \\ 5x &= 115 \\ x &= 23 \end{aligned}$$



$$\begin{aligned} -4x + 9 &= 8x - 3 \\ +4x \quad +3 & \quad +4x \quad +3 \\ 12 &= 12x \\ 1 &= x \end{aligned}$$

3. Use the figure shown.



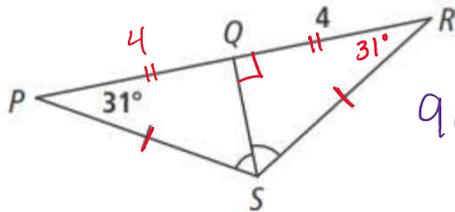
$$8x - 1 = 4x + 5$$

$$\begin{array}{r} -4x \\ \hline -4x + 1 = 4x + 5 - 4x - 1 \\ -4x + 1 - 1 = 4x - 4x + 5 - 1 \\ -4x = 4x - 4 \\ -4x - 4x = 4x - 4x - 4 \\ -8x = -4 \\ x = \frac{-4}{-8} = \frac{1}{2} \end{array}$$

a. What is the value of x ?

b. What are the lengths of all three sides of the triangle?

4. Use the figure shown.



$$90 + m\angle RSQ + 31 = 180$$

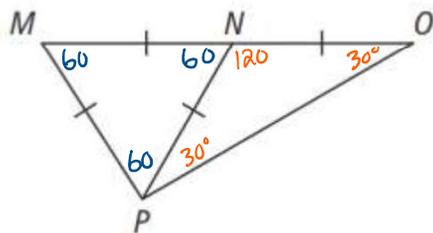
a. What is $m\angle RSQ$?

$$m\angle RSQ = 59^\circ$$

b. What is PR ?

$$PR = 8$$

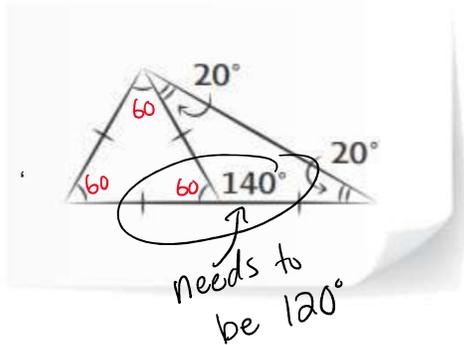
6. Find each angle measure in the figure.



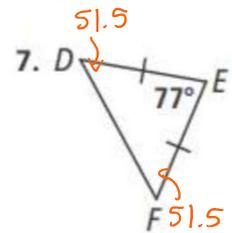
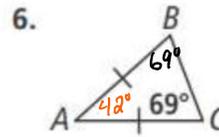
a. $m\angle PNO = 120^\circ$

b. $m\angle NOP = 30^\circ$

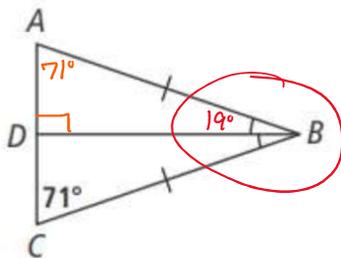
Error Analysis Nate drew the following diagram to represent an equilateral triangle and an isosceles triangle. What mistake did Nate make?



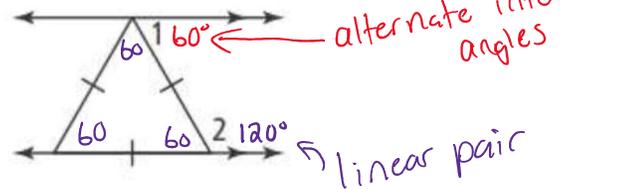
For Exercises 6 and 7, find the unknown angle measures.



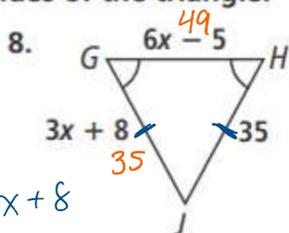
10. What is $m\angle ABD$ in the figure?



12. **Mathematical Connections** What are the measures of $\angle 1$ and $\angle 2$? Explain.



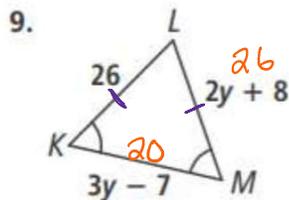
For Exercises 8 and 9, find the lengths of all three sides of the triangle.



$$35 = 3x + 8$$

$$27 = 3x$$

$$9 = x$$



$$26 = 2y + 8$$

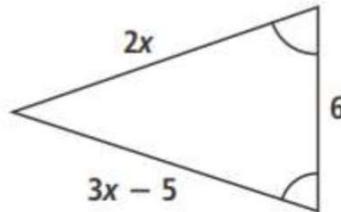
$$18 = 2y$$

$$9 = y$$

Day 3 - CONGRUENCE TRIANGLE POSTULATES

Warm- UP

15. **Error Analysis** Amaya is asked to find the side lengths of the triangle shown. What is her error?



$$2x = 3x - 5$$

From the top leg and the base,
 $2x = 6$, so $x = 3$. Substitute x into
 the expression for the bottom
 leg's length to get $3(3) - 5 = 4$.

X

Warm-Up: Complete the following! It is review for your quiz tomorrow! (:

1. Given $\triangle BAT \cong \triangle GEO$, $\overline{AB} = 4x + 2$, $BT = 6x + 15$, and $GE = 18$. What is the length of \overline{GO} ?

$$\overline{AB} \cong \overline{GE}$$

$$4x + 2 = 18$$

$$4x = 16$$

$$x = 4$$

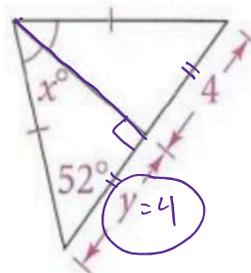
$$\overline{BT} \cong \overline{GO}$$

$$6(4) + 15$$

$$24 + 15$$

$$\overline{GO} = 39$$

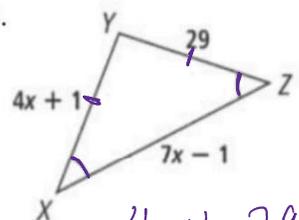
2.



$$90 - 52 = 38$$

$$m\angle x = 38^\circ$$

3.



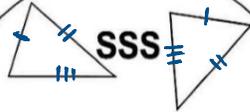
$$4x + 1 = 29$$

$$4x = 28$$

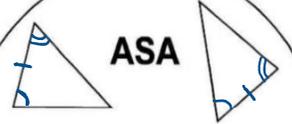
$$x = 7$$

Take out your notes and a calculator! (:

How
Can You
Prove
Triangles
Congruent?



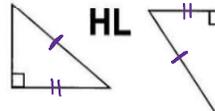
If three sides of one triangle are congruent to three sides of another triangle, then the triangles are congruent.



If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.



If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent.



If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and a leg of a second right triangle, then the triangles are congruent.



If two angles and the non-included side of one triangle are congruent to two angles and the corresponding non-included side of another triangle, then the triangles are congruent.

Graphics Organizer by
Dale Grohman and Linda Meyer
Thomas County Central High School
Thomas County, Georgia

SET

Topic: Triangle Congruence

Determine whether or not the triangles are congruent based on the markings that indicate congruence. For congruent triangles, list the congruence criteria that justifies why the triangles are congruent.

3. *Congruent, SSS*

Two triangles are shown. The first triangle has one side marked with a single tick mark, another with a double tick mark, and the third with a triple tick mark. The second triangle has the same markings on its corresponding sides.

4. *not congruent, AAA*

Two triangles are shown. Each triangle has all three interior angles marked with arcs, indicating they are congruent. The triangles are not similar in shape.

5. *not congruent SSA*

Two triangles are shown. Each has two sides marked with single and double tick marks, and a non-included angle marked with an arc. The triangles are not congruent.

6. *Congruent, SAS*

Two triangles are shown. Each has two sides marked with single and double tick marks, and the included angle between them marked with an arc.

7. *Congruent AAS*

Two triangles are shown. Each has two angles marked with arcs and a non-included side marked with a single tick mark. Red arrows point from the angles to the side.

8. *not congruent SSA*

Two triangles are shown. Each has two sides marked with single and double tick marks, and a non-included angle marked with an arc.

Use the given congruence statement to draw and label two triangles that have the proper corresponding parts congruent to one another.

9. $\triangle ABC \cong \triangle PQR$

Triangle ABC has side AB marked with a single tick mark, side BC with a double tick mark, and side AC with a triple tick mark. Angles A and C are marked with arcs. Triangle PQR has side PQ marked with a single tick mark, side QR with a double tick mark, and side PR with a triple tick mark. Angles P and R are marked with arcs.

10. $\triangle XYZ \cong \triangle KLM$

Triangle XYZ has side XY marked with a single tick mark, side YZ with a double tick mark, and side XZ with a triple tick mark. Angles X and Z are marked with arcs. Triangle KLM has side KM marked with a single tick mark, side ML with a double tick mark, and side KL with a triple tick mark. Angles M and L are marked with arcs.

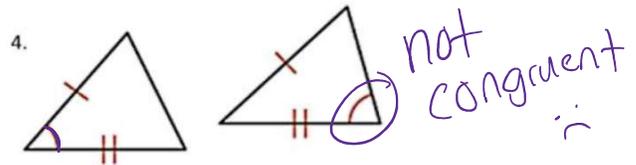
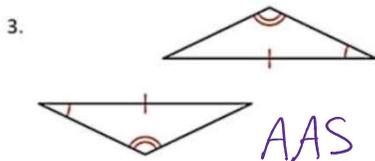
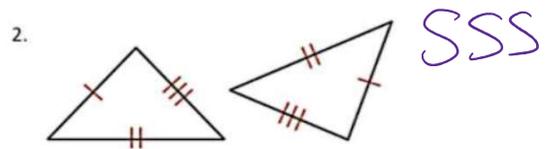
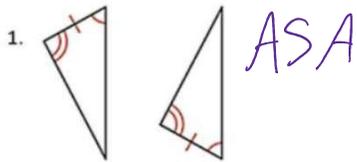
Mathematics Vision Project

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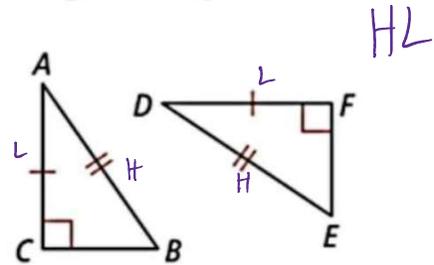
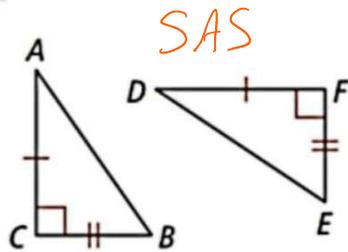
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A. State whether each pair of triangles is congruent by SAS, SSS, ASA, or AAS, or if the congruence cannot be determined.



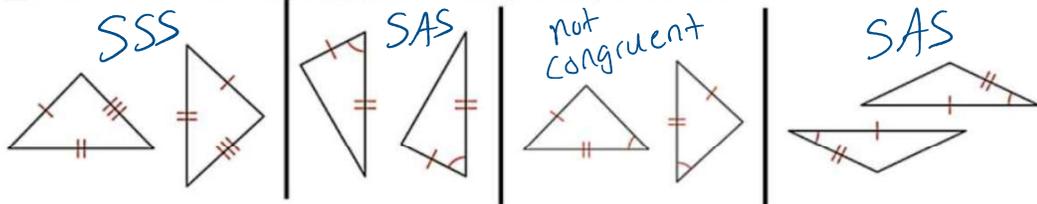
When any two pairs of corresponding sides are congruent, can you show that two right triangles $\triangle ABC$ and $\triangle DEF$ are congruent? Explain.



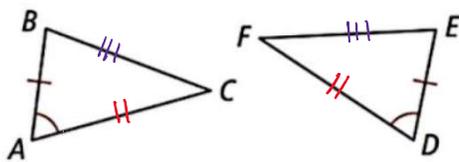
right angles are congruent
to right angles

EXAMPLE 4 Determine Congruent Triangles

A. Which of the following pairs are congruent by SAS or SSS?



B. What additional information is needed to show $\triangle ABC \cong \triangle DEF$ by SAS?
By SSS?



For SAS:
 $\overline{AC} \cong \overline{FD}$

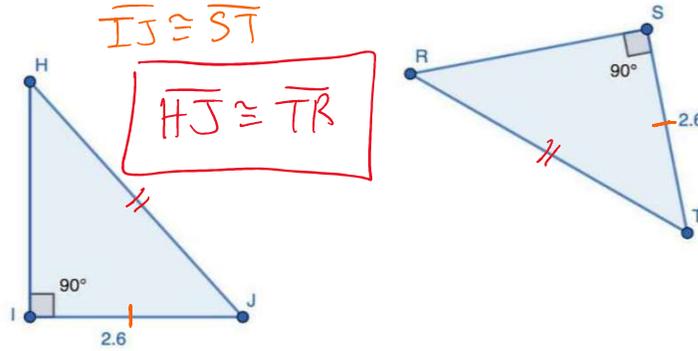
For SSS:
 $\overline{AC} \cong \overline{FD}$,
 $\overline{FE} \cong \overline{BC}$

11. What fact(s) do we need to know in order to say that $\triangle HIJ \cong \triangle RST$ by HL?

Both \triangle 's are right triangles

$$\overline{IJ} \cong \overline{ST}$$

$$\overline{HJ} \cong \overline{TR}$$



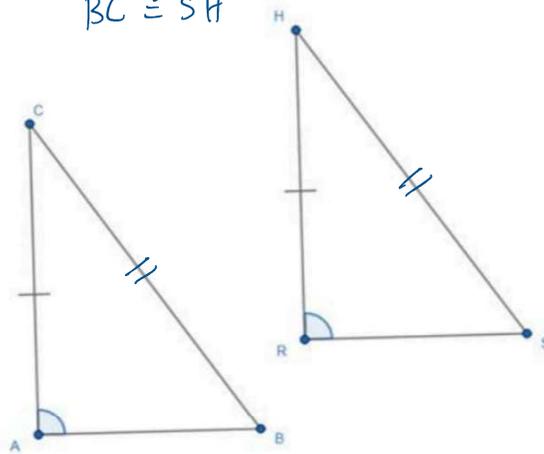
12. What fact(s) do we need to know in order to say $\triangle CAB \cong \triangle HRS$

By ASA criteria: $\angle C \cong \angle H$

By AAS criteria: $\angle S \cong \angle B$

By HL criteria: $\angle A$ and $\angle R$ are right \angle 's.

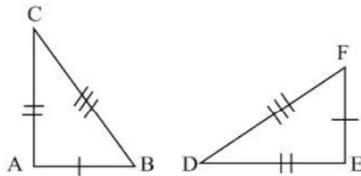
$$\overline{BC} \cong \overline{SH}$$



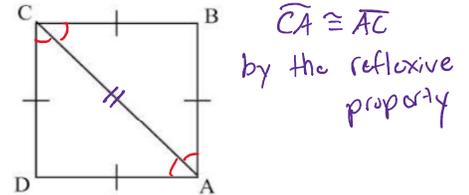
Triangle Congruence Shortcuts Worksheet #1

For each pair of triangles, tell which postulates, **if any**, make the triangles congruent.

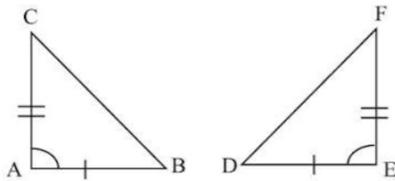
12. $\triangle ABC \cong \triangle EFD$ SSS



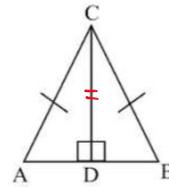
13. $\triangle ABC \cong \triangle CDA$ SAS



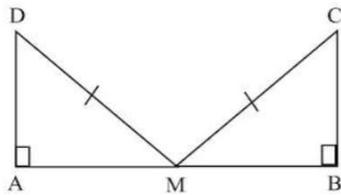
14. $\triangle ABC \cong \triangle EFD$ SAS



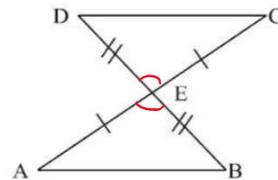
15. $\triangle ADC \cong \triangle BDC$ HL



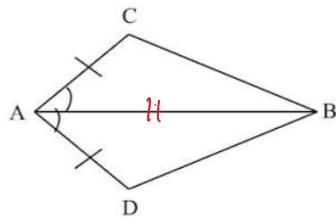
21. $\triangle MAD \cong \triangle MBC$ Not enough info



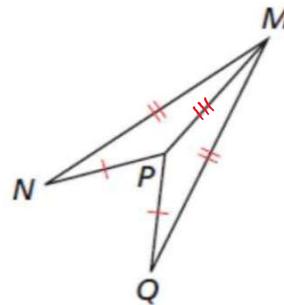
$\triangle ABE \cong \triangle CDE$ SAS



23. $\triangle ACB \cong \triangle ADB$ SAS



23. $\triangle MNP \cong \triangle MQP$ SSS

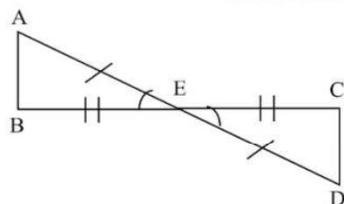


23. SAS

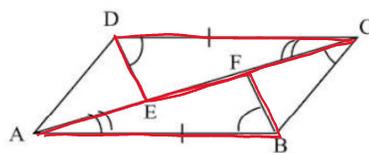
Triangle Congruence Shortcuts Worksheet #2

I. For each pair of triangles, tell which postulate, if any, can be used to prove the triangles congruent.

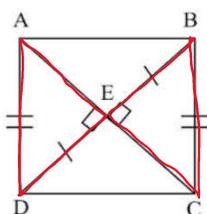
1. $\triangle AEB \cong \triangle DEC$ SAS



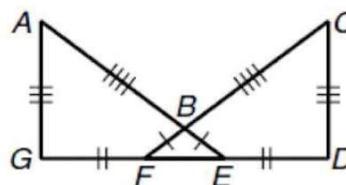
2. $\triangle CDE \cong \triangle ABF$ ASA



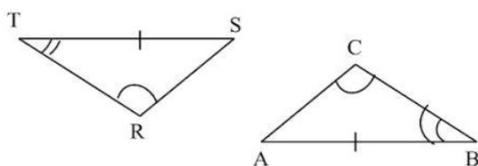
3. $\triangle DEA \cong \triangle BEC$ HL



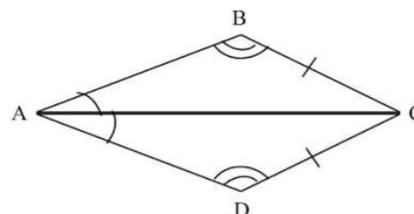
4. $\triangle AGE \cong \triangle CDF$ SSS



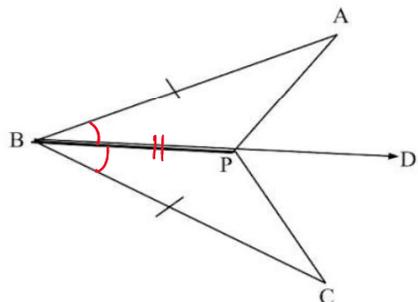
5. $\triangle RTS \cong \triangle CBA$ AAS



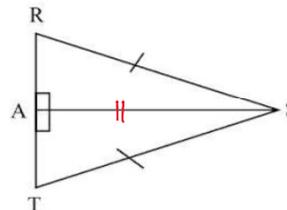
6. $\triangle ABC \cong \triangle ADC$ AAS



7. $\triangle BAP \cong \triangle BCP$ SAS
Given: \overline{BD} bisects $\angle ABC$

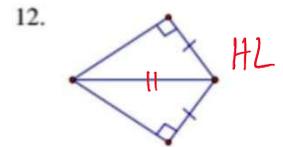
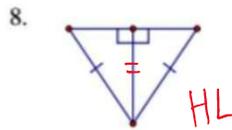
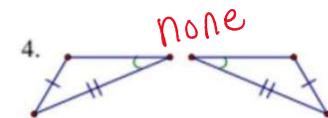
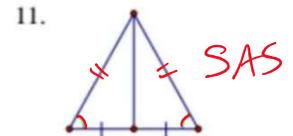
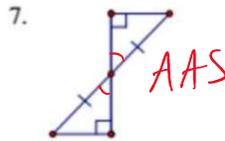
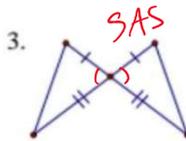
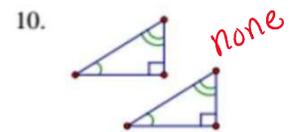
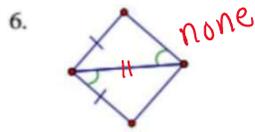
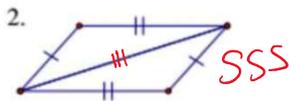
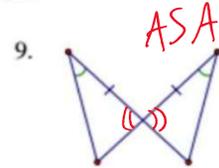
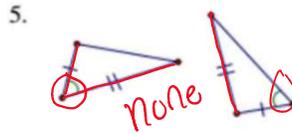
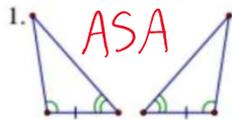


8. $\triangle SAT \cong \triangle SAR$ HL



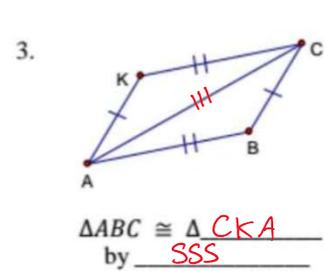
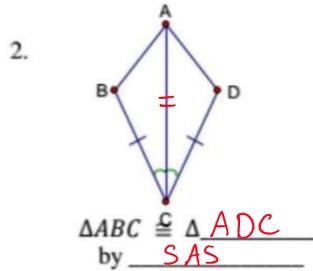
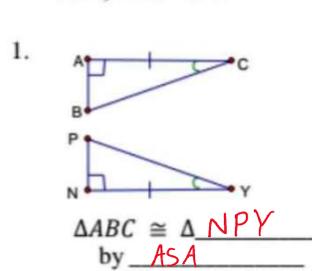
Unit 2- Triangle Congruence Postulates

I. If the triangles can be proven congruent, give the reason (SSS, SAS, ASA, or AAS). If there is not enough information to prove the triangles congruent, write "none."

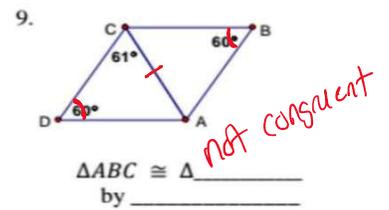
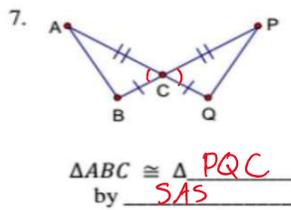
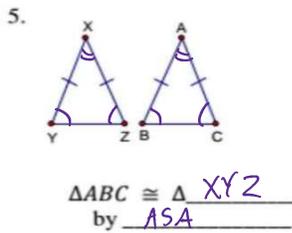
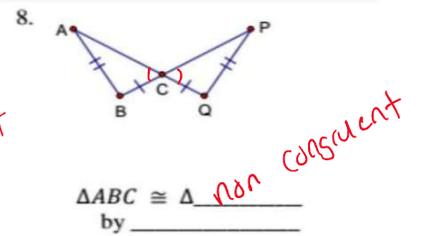
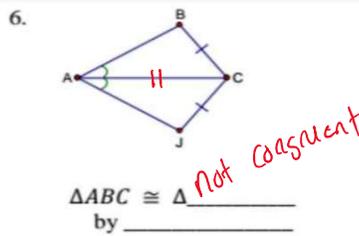
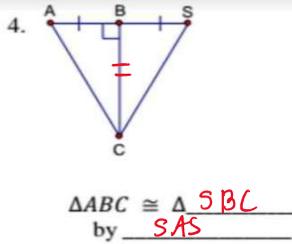


II. Determine whether you can conclude that another triangle is congruent to $\triangle ABC$.

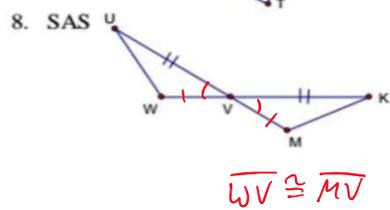
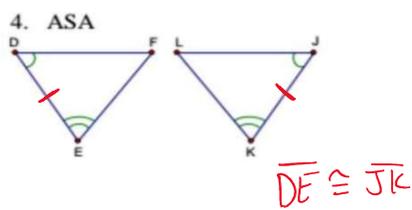
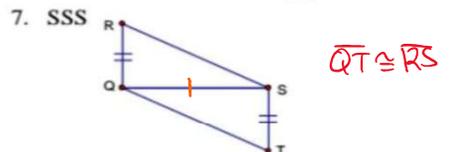
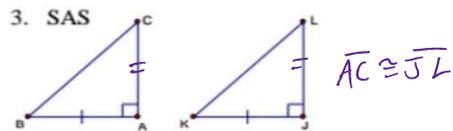
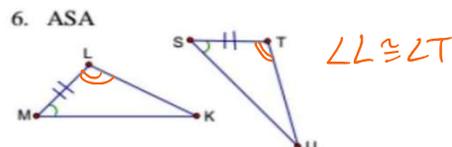
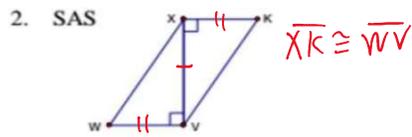
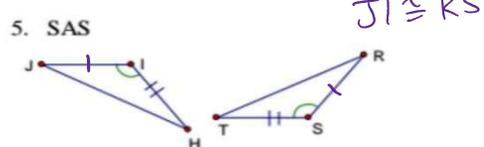
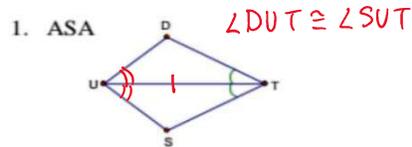
- If so, complete the congruence statement and give the reason (SSS, SAS, ASA, or AAS).
- If not, write "none."



Day 4- CONGRUENCE TRIANGLE POSTULATES and START PROOFS



What additional information is required in order to know that the triangles are congruent by the given reason?

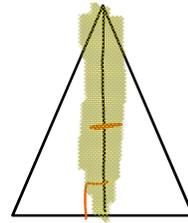


Congruence Proofs

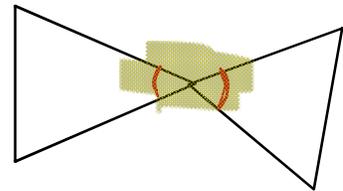
- Label everything on your picture!
 - Start with the given !
 - Add your own sides/angles as you go!

We must look for:

→ reflexive property



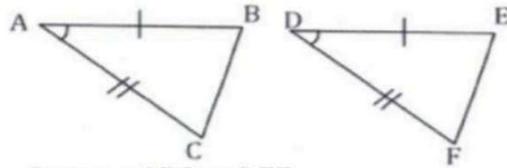
→ vertical angles



- Statement/reasoning: We need to explain our thinking process! Every step must have a reason!
 - Start with the given, our reason in the table will simply be given.
 - State all congruent sides or angles and how we know!
 - End with the prove statement.
 - Proving Triangles are Congruent: We must also say which congruence postulate allowed us to say that!
SSS, ASA, AAS, SAS, HL
 - Proving Sides or Angles are Congruent: We will use CPCTC !

Corresponding parts of congruent triangles are congruent.

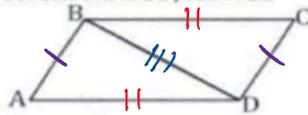
Given: $\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$, and $\angle A \cong \angle D$



Prove: $\triangle ABC \cong \triangle DEF$

Statements	Reasons
1. $\overline{AB} \cong \overline{DE}$	1. Given
2. $\overline{AC} \cong \overline{DF}$	2. Given
3. $\angle A \cong \angle D$	3. Given
4. $\triangle ABC \cong \triangle DEF$	4. SAS

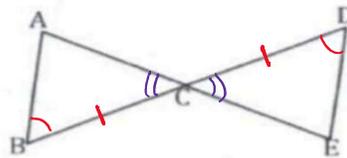
Given: $\overline{AB} \cong \overline{CD}$, $\overline{AD} \cong \overline{CB}$



Prove: $\triangle ABD \cong \triangle CBD$

Statements	Reasons
1. $\overline{AB} \cong \overline{CD}$	1. Given
2. $\overline{AD} \cong \overline{CB}$	2. Given
3. $\overline{BD} \cong \overline{BD}$	3. reflexive property
4. $\triangle ABD \cong \triangle CBD$	4. SSS

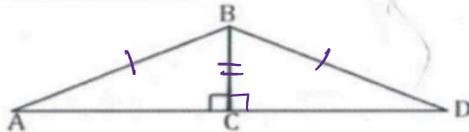
Given: \overline{AE} Bisects \overline{BD} , $\angle B \cong \angle D$



Prove: $\triangle ABC \cong \triangle DBC$

Statements	Reasons
1. $\angle B \cong \angle D$	1. given
2. \overline{AC} Bisects \overline{BD}	2. given
3. $\overline{BC} \cong \overline{DC}$	3. def of line bisector
4. $\angle ACB \cong \angle DCE$	4. vertical \angle 's
5. $\triangle ABC \cong \triangle DBC$	5. ASA

Given: $\overline{AB} \cong \overline{BD}$



Prove: $\triangle ABC \cong \triangle DBC$

Statements	Reasons
1. $\overline{AB} \cong \overline{BD}$	1. given
2. $\overline{BC} \cong \overline{BC}$	2. reflexive property
3. $\triangle ABC \cong \triangle DBC$	3. HL

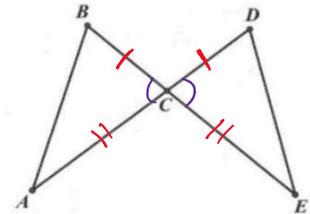
Day 5- PROOFS

Geometry, Unit 5 - Congruent Triangles Proof Activity - Part I Name _____

For each problem, do the following:

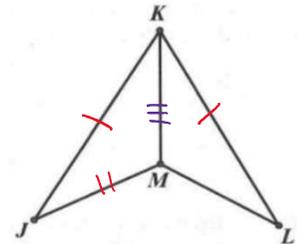
- Show the given information in the diagram (using tick marks to show congruent sides and arcs to show congruent angles)
- Show any other congruent parts you notice (from vertical angles, sides shared in common, or alternate interior angles with parallel lines)
- Give the postulate or theorem that proves the triangles congruent (SSS, SAS, ASA, AAS, HL)
- Finally, fill in the blanks to complete the proof.

1. Given: $\overline{BC} \cong \overline{DC}$; $\overline{AC} \cong \overline{EC}$
 Prove: $\triangle BCA \cong \triangle DCE$



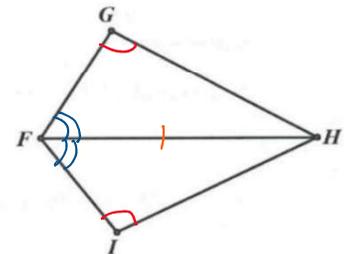
Statements	Reasons
1. $\overline{BC} \cong \overline{DC}$; $\overline{AC} \cong \overline{EC}$	1. Given
2. $\angle BCA \cong \angle DCE$	2. Vertical \angle s Theorem
3. $\triangle BCA \cong \triangle DCE$	3. SAS

2. Given: $\overline{JK} \cong \overline{LK}$; $\overline{JM} \cong \overline{LM}$
 Prove: $\triangle KJM \cong \triangle KLM$



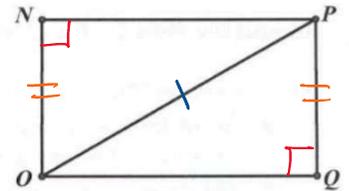
Statements	Reasons
1. $\overline{JK} \cong \overline{LK}$; $\overline{JM} \cong \overline{LM}$	1. Given
2. $\overline{KM} \cong \overline{KM}$	2. Reflexive Prop.
3. $\triangle KJM \cong \triangle KLM$	3. SSS

3. Given: $\angle G \cong \angle I$; \overline{FH} bisects $\angle GFI$
 Prove: $\triangle GFH \cong \triangle IFH$



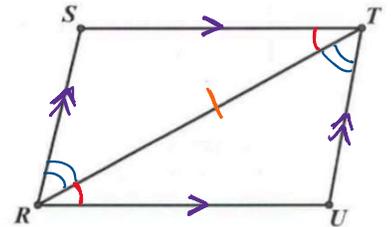
Statements	Reasons
1. $\angle G \cong \angle I$; \overline{FH} bisects $\angle GFI$	1. Given
2. $\angle GFH \cong \angle IFH$	2. Def. of <u>angle bisector</u>
3. $\overline{FH} \cong \overline{FH}$	3. Reflexive Prop.
4. $\triangle GFH \cong \triangle IFH$	4. AAS

4. Given: $\angle N$ and $\angle Q$ are right angles; $\overline{NO} \cong \overline{PQ}$
 Prove: $\triangle ONP \cong \triangle PQO$



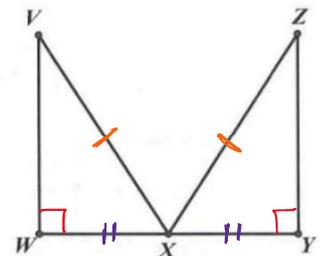
Statements	Reasons
1. $\angle N$ and $\angle Q$ are right angles	1. Given
2. $\triangle ONP$ and $\triangle PQO$ are right triangles	2. Def. of right triangle
3. $\overline{OP} \cong \overline{PO}$	3. Reflexive Prop.
4. $\overline{NO} \cong \overline{PQ}$	4. Given
5. $\triangle ONP \cong \triangle PQO$	5. HL

5. Given: $\overline{ST} \parallel \overline{RU}$; $\overline{SR} \parallel \overline{TU}$
 Prove: $\triangle SRT \cong \triangle UTR$



Statements	Reasons
1. $\overline{ST} \parallel \overline{RU}$	1. Given
2. $\angle STR \cong \angle URT$	2. If lines \parallel , alt. int. \angle s \cong
3. $\overline{SR} \parallel \overline{TU}$	3. Given
4. $\angle SRT \cong \angle UTR$	4. alt int \angle 's are \cong
5. $\overline{RT} \cong \overline{TR}$	5. Reflexive property
6. $\triangle SRT \cong \triangle UTR$	6. ASA

6. Given: $\angle W$ and $\angle Y$ are right angles; $\overline{VX} \cong \overline{ZX}$; X is the midpoint of \overline{WY}
 Prove: $\triangle VWX \cong \triangle ZYX$



Statements	Reasons
1. $\angle W$ and $\angle Y$ are right angles	1. Given
2. $\triangle VWX$ and $\triangle ZYX$ are right \triangle	2. Def. of right triangle
3. $\overline{VX} \cong \overline{ZX}$; X is the midpoint of \overline{WY}	3. Given
4. $\overline{WX} \cong \overline{XY}$	4. Def. of midpoint
5. $\triangle VWX \cong \triangle ZYX$	5. HL

Taking Proofs a Step Further!

We can also use another property!

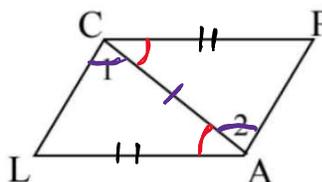
CPCTC: Congruent Parts of Congruent Triangles are Congruent

- We can use this to prove that ∠'s or sides are congruent.
- Before we can do this, we must prove triangles are congruent first.

Given: $\angle LAC \cong \angle PCA$

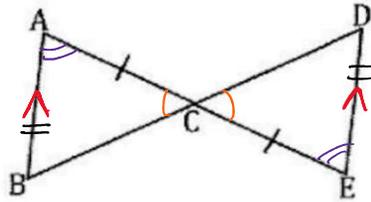
$\angle 1 \cong \angle 2$

Prove: $\overline{LA} \cong \overline{CP}$



Statement	Reasons
1. $\angle LAC \cong \angle PCA$	1. Given
2. $\angle 1 \cong \angle 2$	2. Given
3. $\overline{CA} \cong \overline{AC}$	3. Reflexive property
4. $\triangle LAC \cong \triangle PCA$	4. ASA
5. $\overline{LA} \cong \overline{CP}$	5. CPCTC

Given $\overline{AB} \parallel \overline{ED}$, $\overline{AC} \cong \overline{EC}$



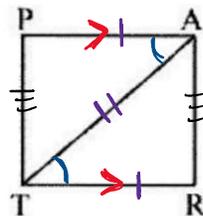
Prove: $\overline{AB} \cong \overline{ED}$

Statement	Reasons
1. $\overline{AB} \parallel \overline{ED}$	1. Given
2. $\overline{AC} \cong \overline{EC}$	2. Given
3. $\angle ACB \cong \angle ECD$	3. Vertical \angle 's
4. $\angle BAC \cong \angle DEC$	4. Alt. int. \angle 's
5. $\triangle ACB \cong \triangle ECD$	5. ASA
6. $\overline{AB} \cong \overline{ED}$	6. CPCTC

Given: $\overline{PA} \parallel \overline{TR}$

$\overline{PA} \cong \overline{TR}$

Prove: $\overline{PT} \cong \overline{AR}$



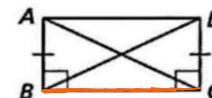
Statement	Reasons
1. $\overline{PA} \parallel \overline{TR}$	1. Given
2. $\overline{PA} \cong \overline{TR}$	2. Given
3. $\angle PAT \cong \angle RTA$	3. Alt. int. \angle 's
4. $\overline{AT} \cong \overline{TA}$	4. Reflexive property
5. $\triangle PAT \cong \triangle RTA$	5. SAS
6. $\overline{PT} \cong \overline{AR}$	6. CPCTC

Day 6 OVERLAPPING TRIANGLES

9-6 Overlapping Triangles (V2: 295 - 302)

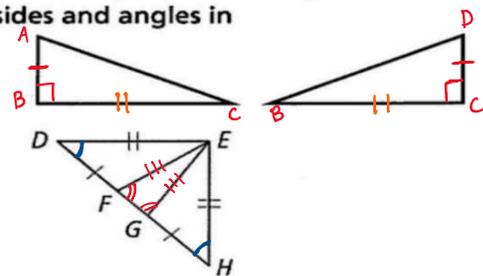
EXAMPLE 1 Identify Corresponding Parts in Triangles

Figure $ABCD$ is a rectangle with diagonals \overline{AC} and \overline{BD} . Why is it important to identify corresponding parts of overlapping triangles?



Consider $\triangle ABC$ and $\triangle DCB$. Identify the corresponding sides and angles in the two triangles by first determining congruent parts.

(Complete Try It 1 on Pg. 296)



EXAMPLE 2 Use Common Parts of Triangles

Is $\angle EGD \cong \angle EFH$? YES \rightarrow

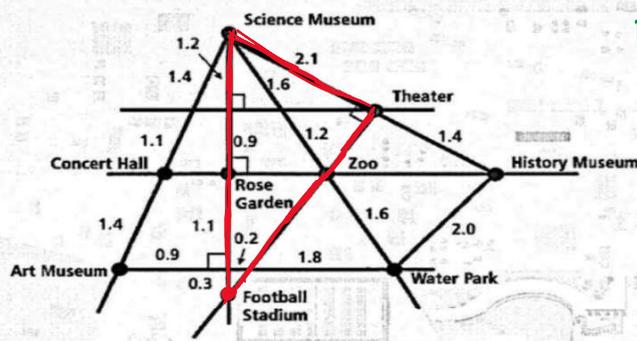
$\triangle FEG$ is isosceles so,
 $\angle EGD \cong \angle EFH$

(Complete Try It 2 on Pg. 296)

(Complete Try It 3 on Pg. 297)

EXAMPLE 4 Separate Overlapping Triangles

A city runs three triangular bus routes to various attractions. How can you draw a separate triangle for each route? Are any of the routes the same length?



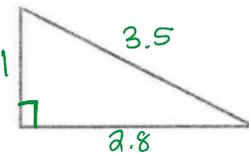
$$(2.1)^2 + b^2 = (3.5)^2$$

$$4.41 + b^2 = 12.25$$

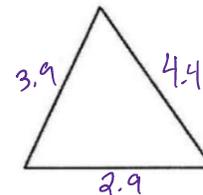
$$b^2 = 7.84$$

$$b = 2.8$$

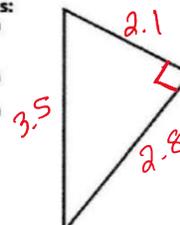
Green Route



Purple Route



Red Route



Green Route Stops:
Science Museum
Theater
History Museum
Zoo
Rose Garden
Science Museum

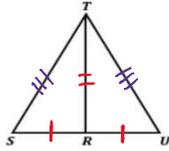
Purple Route Stops:
Water Park
Art Museum
Concert Hall
Science Museum
Zoo
Water Park

Red Route Stops:
Football Stadium
Zoo
Theater
Science Museum
Rose Garden
Football Stadium

(Complete Try It 4 on Pg. 297)

Green and red are same length.

Segment RT bisects the base, side SU , of isosceles triangle STU .



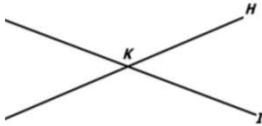
The table shows the steps to prove $\angle STR \cong \angle UTR$.

Step	Statement	Reason
1	RT bisects side SU .	Given
2	$SR \cong UR$	Definition of segment bisector
3	$RT \cong RT$?
4	$ST \cong UT$	Definition of isosceles triangle
5	$\triangle STR \cong \triangle UTR$?
6	$\angle STR \cong \angle UTR$	Corresponding parts of congruent triangles are congruent.

Which reasons are needed to complete the proof?

- A Reason 3: Reflexive Property
Reason 5: Side-Angle-Side postulate
- B Reason 3: Reflexive Property
Reason 5: Side-Side-Side postulate
- C Reason 3: Symmetric Property
Reason 5: Side-Angle-Side postulate
- D Reason 3: Symmetric Property
Reason 5: Side-Side-Side postulate

segment GI intersects segment JH at point K . The table outlines a proof showing that $\angle H \cong \angle JKI$.



Statement	Justification
Segment GI intersects segment JH at point K .	Given
$m\angle GKH + m\angle HKI = 180^\circ$	Definition of linear pair
$m\angle JKI + m\angle HKI = 180^\circ$	Definition of linear pair
$m\angle GKH + m\angle HKI = m\angle JKI + m\angle HKI$	Substitution Property
$\angle GKH \cong \angle JKI$	Definition of congruent angles

Which statement and justification best fill in the blanks to complete the proof that vertical angles are congruent?

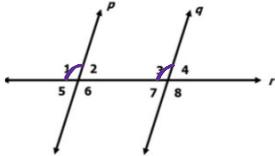
A $m\angle GKH = m\angle JKI$; Subtraction Property

B $m\angle GKH = m\angle JKI$; Commutative Property

C $m\angle GKH = m\angle HKI$; Transitive Property

D $m\angle GKH = m\angle HKI$; Addition Property

Parallel lines p and q are cut by transversal r .



A student starts a proof to show that alternate interior angles $\angle 2$ and $\angle 7$ are congruent.

Statement	Justification
Lines p and q are parallel.	Given
$\angle 1 \cong \angle 3$?
$m\angle 1 = m\angle 3$	Definition of congruence
$m\angle 1 + m\angle 2 = 180^\circ$ and $m\angle 3 + m\angle 7 = 180^\circ$?
$m\angle 1 + m\angle 2 = 180^\circ$	Substitution Property
$m\angle 7 = 180^\circ - m\angle 1$ and $m\angle 2 = 180^\circ - m\angle 1$?
$m\angle 2 = m\angle 7$	Substitution Property
$\angle 2 \cong \angle 7$	Definition of congruence

Which justifications, in order, complete the proof?

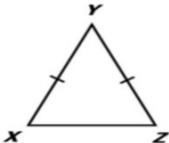
A Alternate Interior Angles Theorem, definition of a linear pair, and Distributive Property

B Alternate Interior Angles Theorem, definition of a linear pair, and Subtraction Property

C Corresponding Angles Theorem, definition of a linear pair, and Distributive Property

D Corresponding Angles Theorem, definition of a linear pair, and Subtraction Property

Triangle XYZ is an isosceles triangle.



The table shows the steps used to prove that the base angles of triangle XYZ , $\angle X$ and $\angle Z$, are congruent.

Step	Explanation
1	Triangle XYZ is isosceles, therefore $XY = YZ$.
2	Use the Reflexive Property to show that $WY = WY$, and therefore $\triangle XWY \cong \triangle ZWY$ by the Side-Side-Side Postulate.
3	Plot point W at the midpoint of line segment XZ .
4	Since $\triangle XWY \cong \triangle ZWY$, $\angle X \cong \angle Z$ because corresponding parts of congruent triangles are congruent.
5	Since point W is the midpoint of XZ , $XW = WZ$.

What is the correct order of the steps?

A 1, 3, 2, 5, 4

B 1, 3, 2, 4, 5

C 1, 3, 5, 4, 2

D 1, 3, 5, 2, 4