Unit 1: Rigid Motion Transformations

Unit 1 Main Objectives:

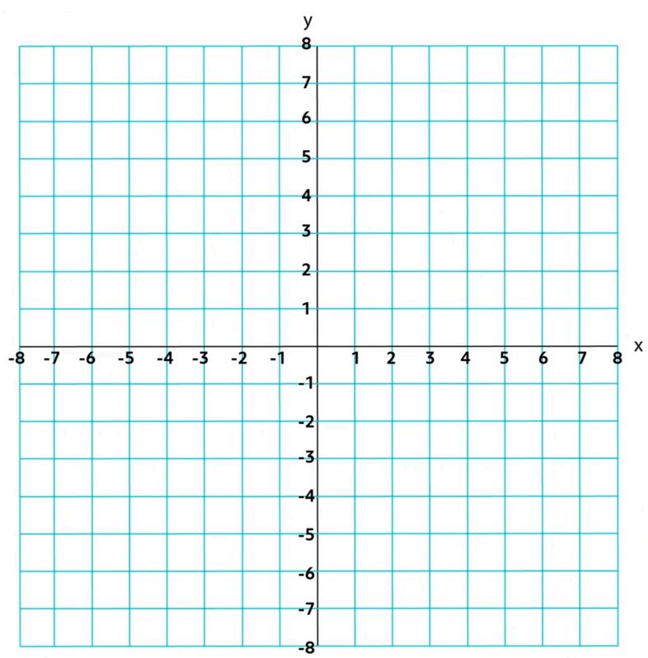
- ☐ I can use geometric descriptions of rigid motions to transform figures.
- ☐ I can write and explain rules for rigid motions in formal notation.
- ☐ I can identify rotational and reflectional symmetry in figures.

Key Vocabulary: I can define the following vocabulary terms.

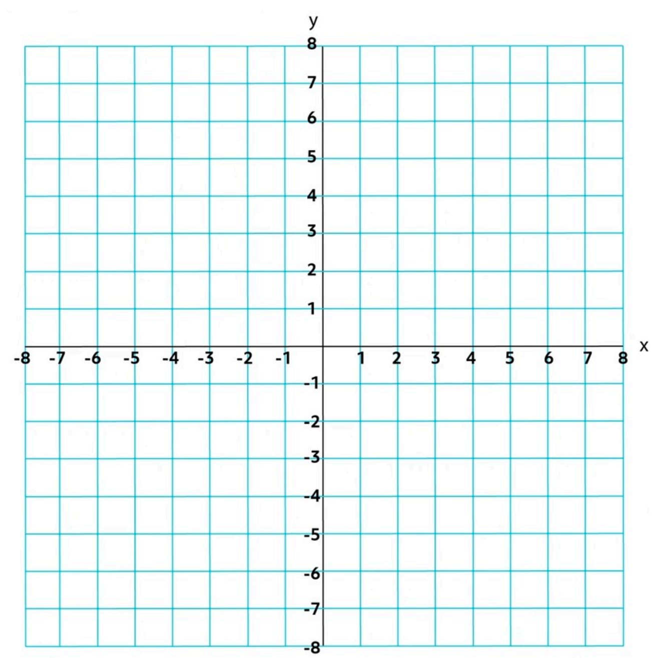
- ☐ Image
- ☐ Preimage
- ☐ Composition
- ☐ Transformation
- ☐ Rigid Motion
- ☐ Translation
- ☐ Formal Notation
- ☐ Reflection
- ☐ Line of reflection
- ☐ Equidistant
- ☐ Perpendicular Bisector
- ☐ Line of Symmetry
- ☐ Rotational Symmetry
- ☐ Glide Reflection
- ☐ Rotation
- ☐ Center
- ☐ Degrees
- ☐ Clockwise
- ☐ Counterclockwise
- ☐ Parallel
- ☐ Congruent

Day 0 - SKILLS CHECK

1. Label the origin, the axes, and the quadrants of the coordinate plane.



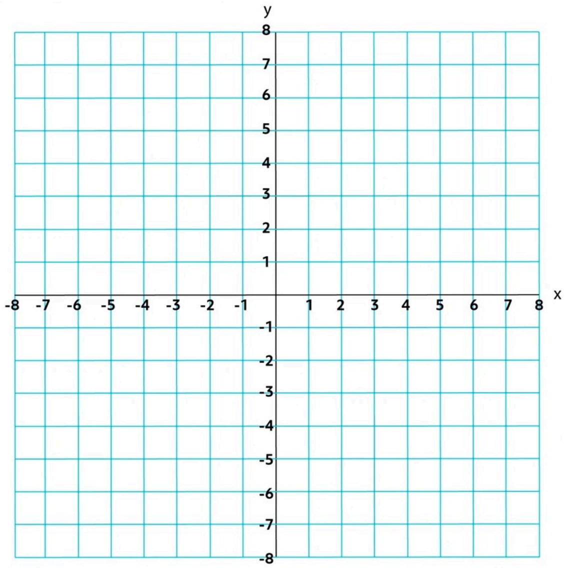
2. Graph and label the lines $y = 3$ and $x = -2$.



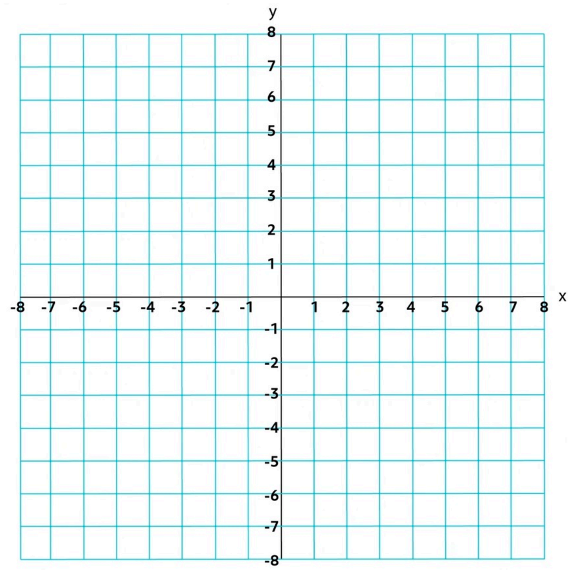
3. Using Slope formula, find the slope between (-3,-2) and (4,1).

4. What slope is perpendicular to a slope of 4?

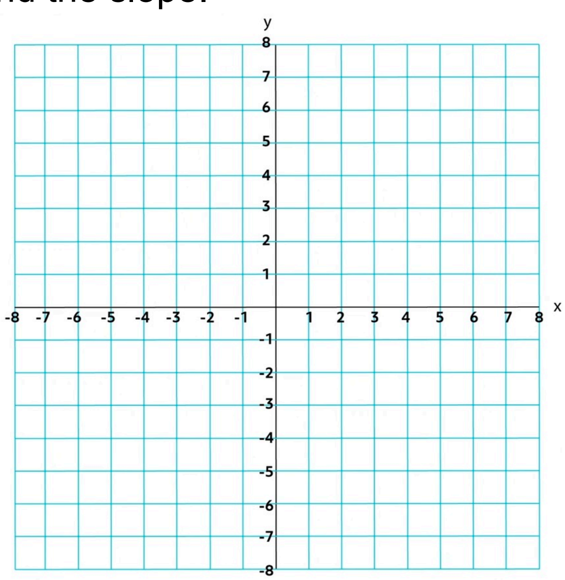
5. Plot the points A (-3, 4), B (0, 3), C (1,0), D (2, 5), and E (3, -4)



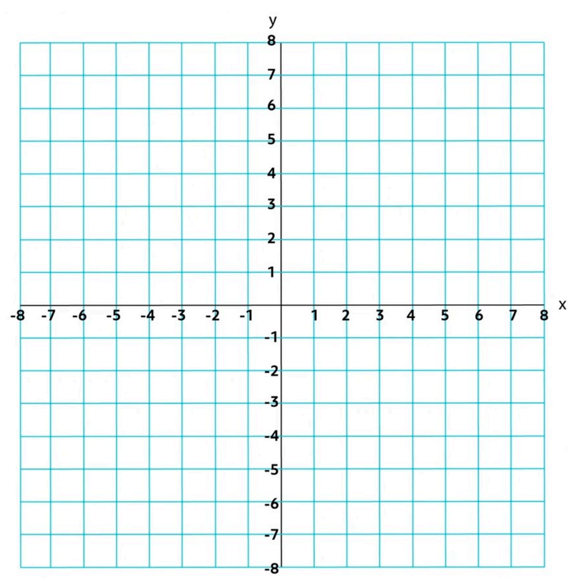
6. Graph the line $y = x$



7. Plot the points A(-3,4) and B(3,2) and find the slope.



8. Graph the line $y =- x$



Day 0 - RIGID MOTION

- A Transformation is a way of changing the ____________ or ______________ of a shape.

- Rigid Motions: __________________ and ________________ measures are

  preserved. We can think of this as being the same size and shape, or

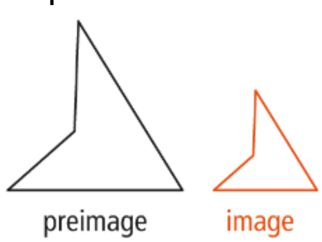
  ____________________ figures.

    - The three rigid motion transformations include:
        - Translations
        - Rotations
        - Reflection

- A dilation is another transformation we will discuss later this semester in Math 2!

  Dilations change the ____________ of a shape.

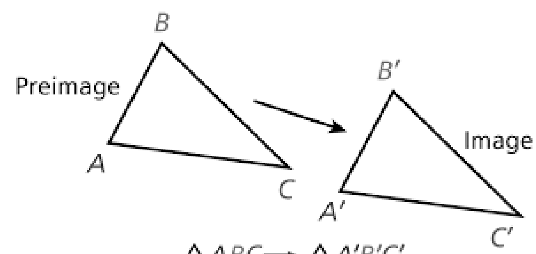
Example 1: Is the transformation a rigid motion? Explain?



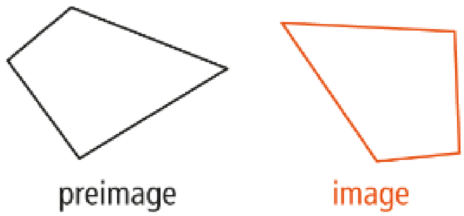
- Preimage - is the ___________________ figure. The "____________"

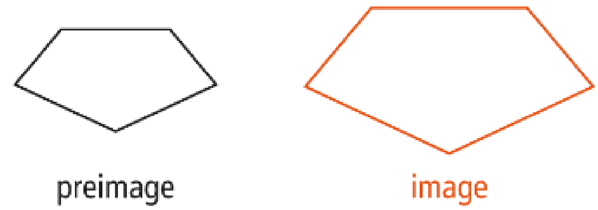
- Image - is the ___________________ figure. The "____________"

Notice the labels on each shape. What do you see?



You Try! Are the following transformations a rigid motion? Explain

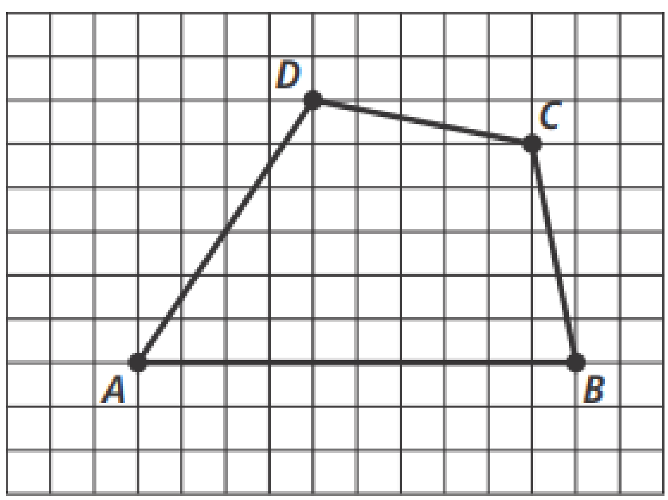




Day 1 - TRANSLATIONS (LESSON 8-2)

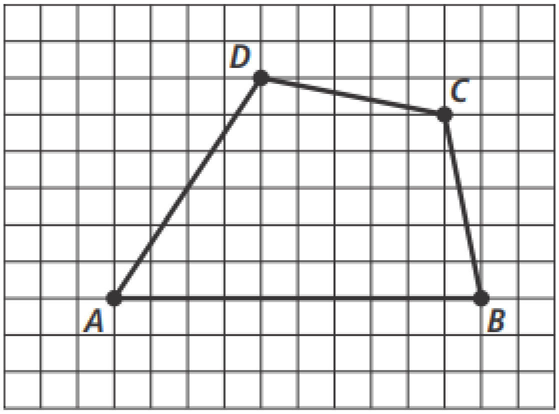
EXPLORE & REASON

Draw a copy of *ABCD* on a grid. Using another color, draw a copy of *ABCD* on the grid in a different location with the same orientation, and label it *QRST*.



Describe below how to move ABCD to the location of QRST.

Without looking at your partner's image. Exchange directions with a partner. Follow your partner's inductions to draw EFGH on the grid below. Do your drawings look the same? Explain.



What makes a set of instructions for this task a "good" set of instructions?

Why is placing the figures on a grid helpful in writing a set of instructions?

You just completed a Translation!

- A translation moves all points of a pre-image in the same __________________

  and______________________. In simple terms we "_____________" the pre-image.

Different types of Notations tell us how to “x” units along the x-axis and “y” units along the y-axis.

Formal Notation: $T_{(x,y)}$

Vector Notation: $< x, y >$

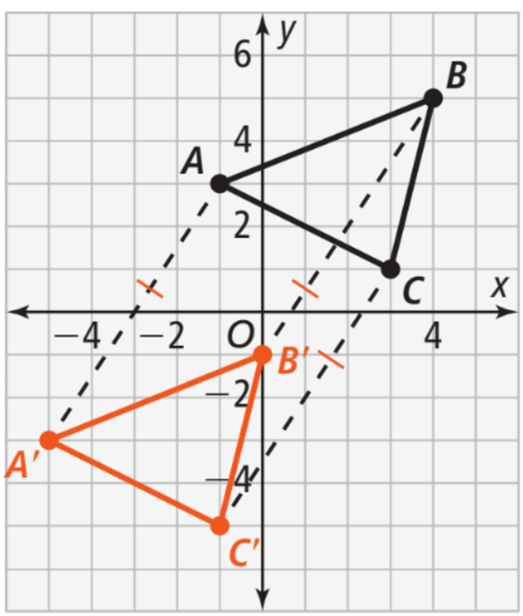
Algebraic Notation: $(x, y) \rightarrow (x \pm \#, y \pm \#)$

Examples:

a. Translate Left 2, Up 8

b. Translate Down 3, Right 6

Label the pre-image and image then describe the translation from $\Delta ABC\ to\ \Delta A'B'C'$



In Words:

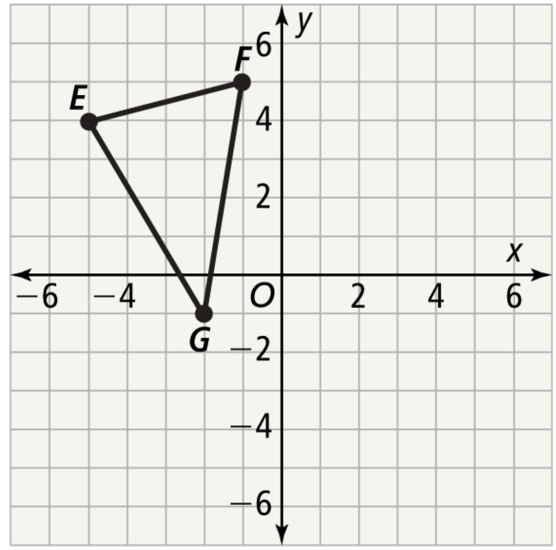
Formal Notation:

Vector Notation:

Algebraic Notation:

Properties of Translations:

Example 1: What is the graph of $T_{<7,-4>}(\triangle EFG) = \triangle E'F'G'$?



- Pre-image Points:

- Image Points:

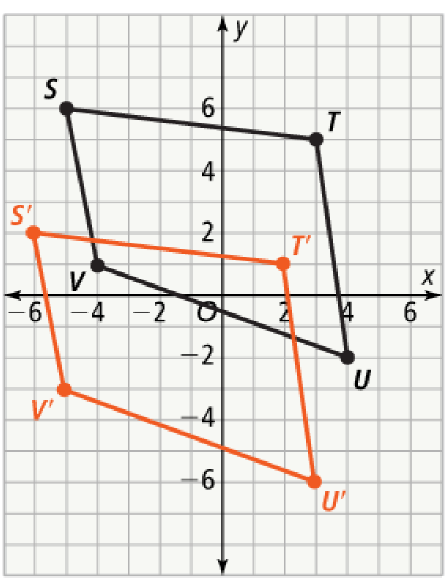
You Try! What are the vertices of △E'F'G for:

a. $T_{<6,-7>}(\triangle EFG) = \triangle E'F'G'$?

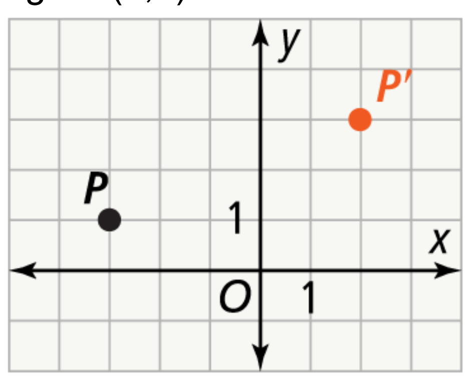
b. $T_{<11,2>}(\triangle EFG) = \triangle E'F'G'$?

Example 2: Write a Translation Rule in all forms.

What translation rule maps STUV onto S'T'U'V'.



Try It! What translation rule maps P(-3,1) to its image P'(2,3).

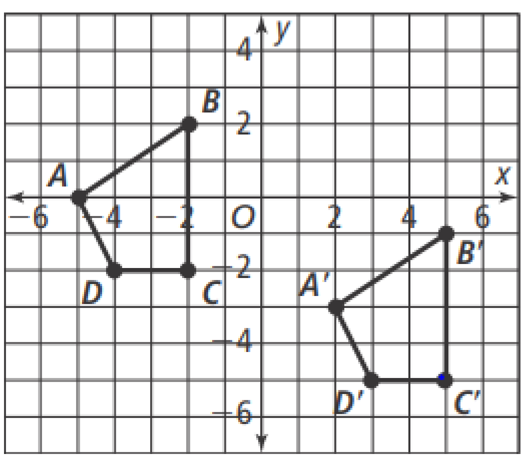


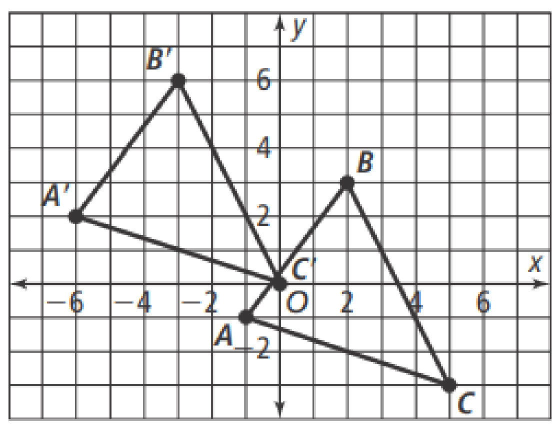
Make Sense and Persevere What are the values of x and y if $T_{\langle -2, 7 \rangle}(x, y) = (3, -1)$?

Practice:

1. For a-b, Describe the transformations below in words:
   a. $T_{<-4,-2>}(\triangle EFG)$ b. $T_{<5,-3>}(\triangle EFG)$

2. For a-b. What is the rule for the transformation shown? Write your answers in at least two forms.





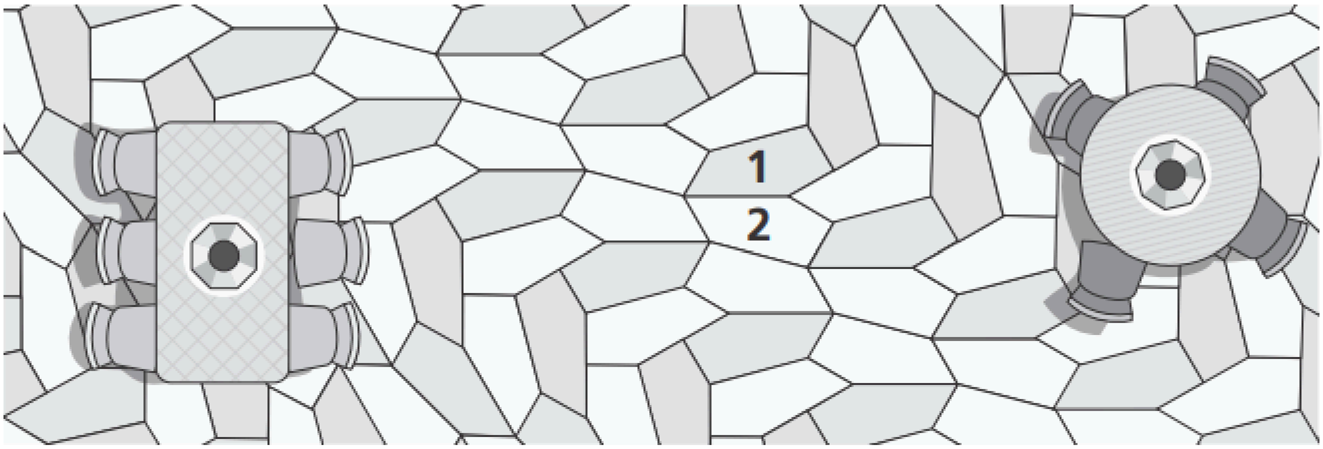
3. For a-b, give the coordinates of the image.
   a. $T_{<3,-1>}(\triangle ABC)$ for $A(5, 0)$, $B(-1, 2)$, $C(6, -3)$

   b. $T_{<-4,0>}(\triangle DEF)$ for $D(3, 3)$, $B(-2, 3)$, $C(0, 2)$

DAY 2 - REFLECTIONS (LESSON 8-1)

EXPLORE & REASON

The illustration shows irregular pentagon-shaped tiles covering a floor.



A. Which tiles are copies of tile 1? Explain.

B. If you were to move tile 1 from the design, what would you have to do so it completely covers tile 2?

- A reflection moves all points of a pre-image across a ______________ _______ ______________________ (LOR) . In simple terms we "_____________" the pre-image. Can also be called a ________________ ____________________.

Formal Notation: $R_{(LOR)}(Shape)$

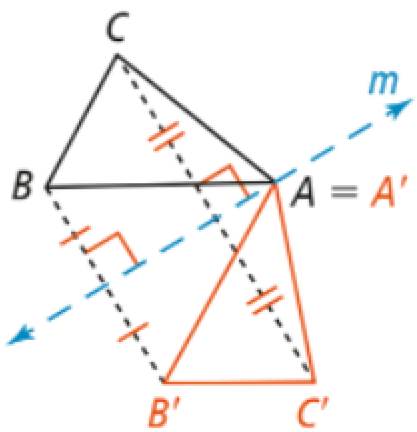
Examples: For a-c Reflect △ABC

a. Over x-axis.

b. Reflect over y=x

c. Reflect over x=5

Label the pre-image and image then describe the reflection from $\Delta ABC\ to\ \Delta A'B'C'$



In Words:

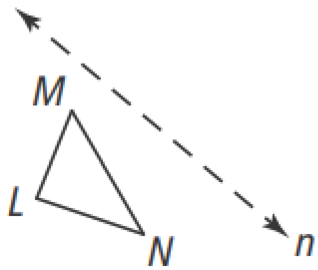
Formal Notation:

Properties of Reflections:

Let's Practice Reflections on the next page!

You Try:

1. What is the reflection of the triangle ΔLMN across line n? Write the Formal Notation.

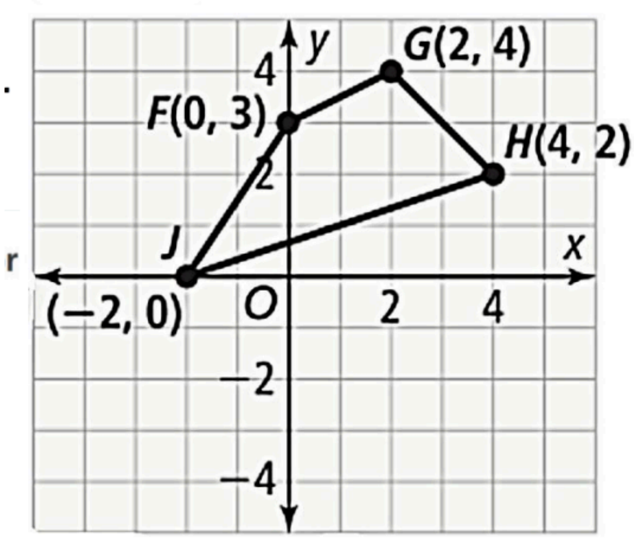


2. Triangle ABC has vertices $A(-5, 6),\ B(1, -2),\ and\ C(-3, -4)$. What are the coordinates of the vertices of Triangle A'B'C' after the following transformations?

a. R_{x-axis}

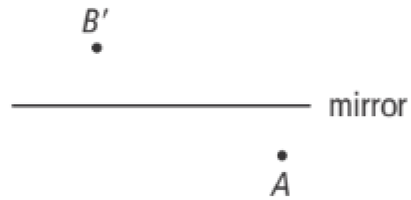
b. $R_{y=x}$

3. Reflect over the y-axis. What do you notice? Why does this happen?

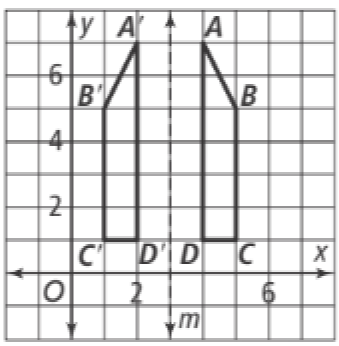


4. What is the reflection rule for the triangle and image with the coordinates $A(2, 4),\ B(4, 6),\ C(5, 2)$ and $A'(-4, -2),\ B'(-6, -4),\ C'(-2, -5)$?

5. Student A sits in a chair facing a mirror and sees the reflection image B’ of Student B in the mirror. Show the actual position of Student B.

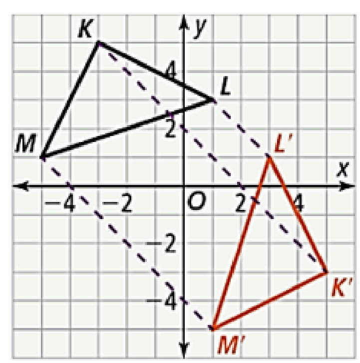


6. The graph shows the reflection of quadrilateral ABCD across line m. The reflection is written $R_m(ABCD) \rightarrow (A'B'C'D')$. What is the equation of line m?



EXAMPLE 4 Describe a Reflection on the Coordinate Plane

What reflection maps $\triangle KLM$ to its image?



Step 1: Write coordinates of pre-image and image.

M (-5,1) M'

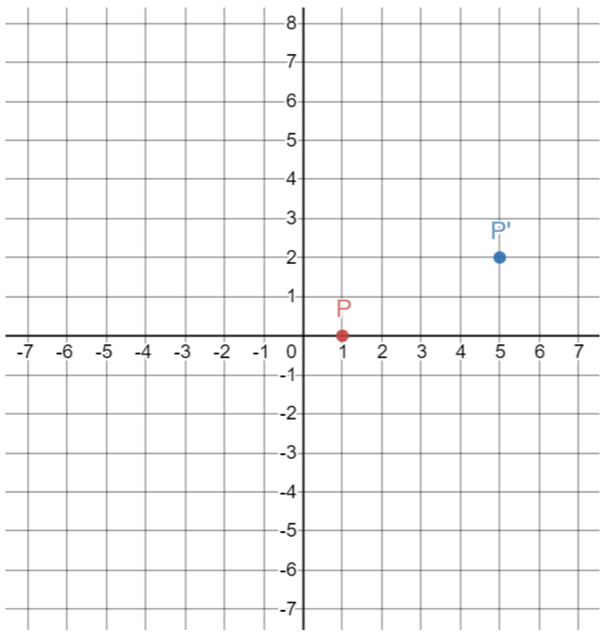
L (1,3) L'

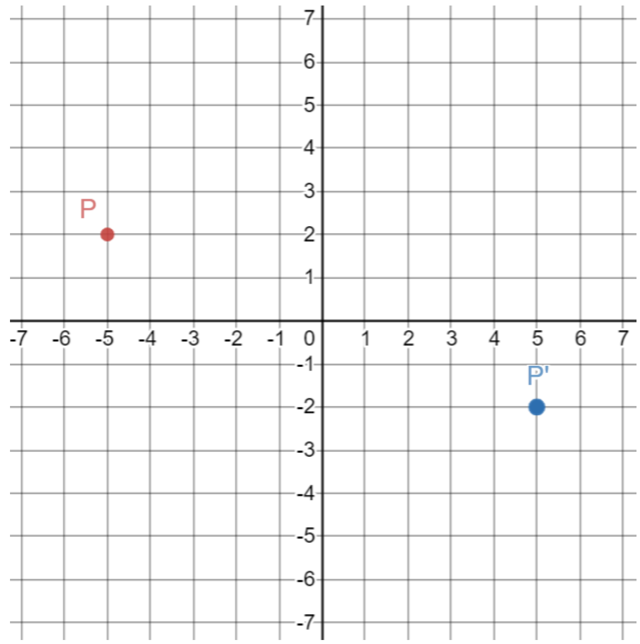
K (-3,5) K'

Step 2: Pick 2 pairs of corresponding points and find the midpoint between each pair.

Midpoint Formula:

Find the Line of Reflection between P and P’ for each of the following graphs.



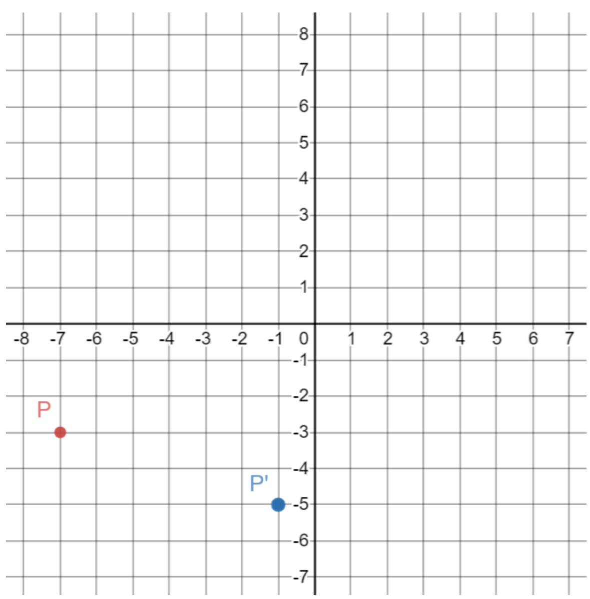


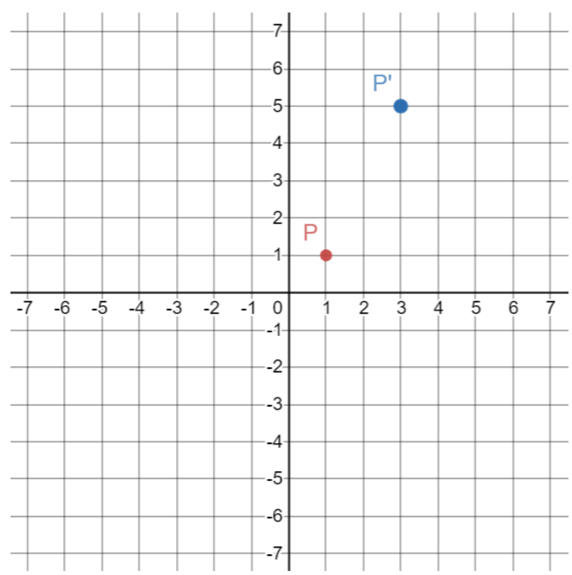
a. Slope: ________ Perp Slope: _______

Equation: y = _______________________

b. Slope: ________ Perp Slope: _______

Equation: y = _______________________





c. Slope: ________ Perp Slope: _______

Equation: y = _______________________

d. Slope: ________ Perp Slope: _______

Equation: y = _______________________

Day 3 - ROTATIONS (Lesson 8-3)

Explore & Reason: Filipe says that the next time one of the hands of the clock points to 7 will be at 7:00, when the hour hand points to 7. Nadia disagrees.

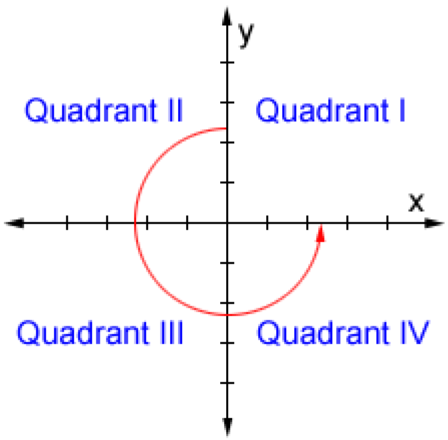
Who do you think is correct? Explain.

Suppose the numbers on the clock face are removed. Write instructions that another person could follow to move the minute hand from 2 to 6.

You just completed a Rotation! How are rotating and translating a figure alike? How are they different?

- A rotation moves all points **about** a point P, called the _______________ _______ __________________, by and angle measured in ________________ called the ___________ ______ ________________, in a specified direction. Either ___________________________ or _____________________________. In simple terms we "_____________" the pre-image.

- The default direction is ______________________________. Because it moves with the quadrants.



How many degrees would each quadrant make up?

Negative Degrees move: __________________________

Do you think a rotated image would ever coincide with the original figure? Explain.

Formal Notation: $r_{(degree,\ center)}(Shape)$

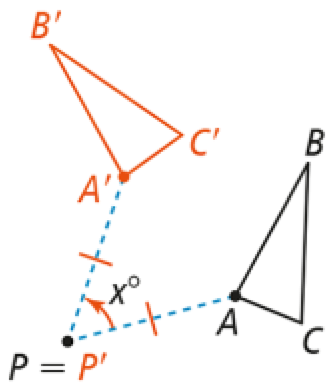
Examples: For a-c Rotate△ABC

a. 90 degrees counterclockwise , about the origin

b. 270 degrees clockwise, about the point (2,-5)

c. 45 degrees, about the origin.

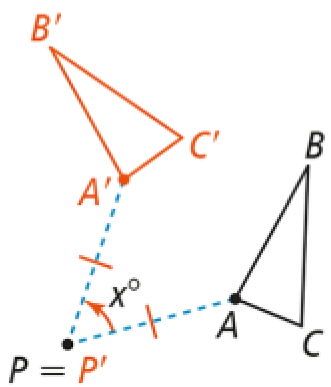
Label the pre-image and image then describe the rotation from $\Delta ABC\ to\ \Delta A'B'C'$



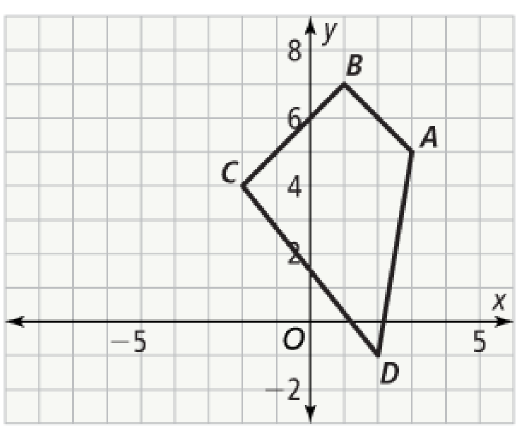
In Words:

Formal Notation:

Properties of Reflections:



Let's try!

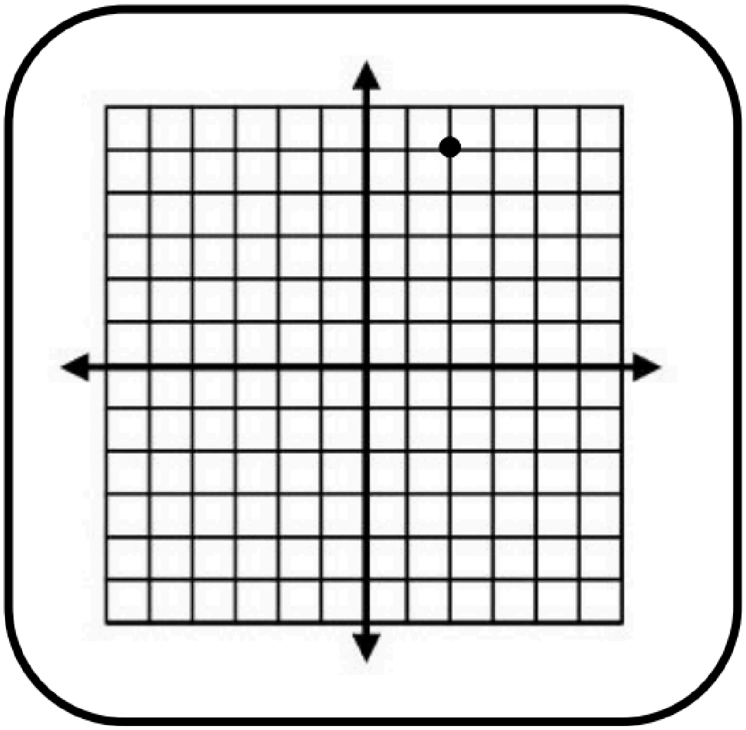


What is $r_{(90,O)}$ ABCD? Explain each part.

Take patty paper and put a plus sign on the center of rotation and turn your patty paper!

How many turns do we need to do for 90 degrees?

Rotations Rules about the Origin:



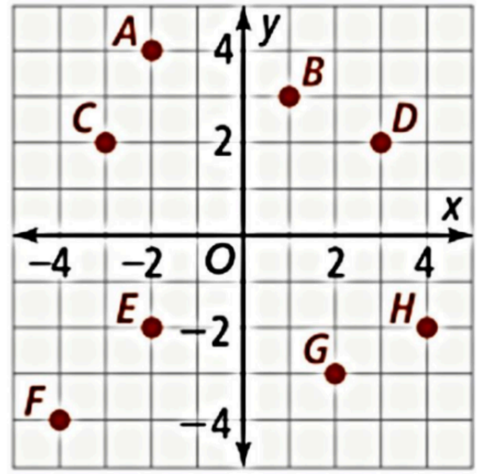
	90° C 270° CC	180° C 180° CC	270° C 90° CC	360° C 360° CC
(x,y)				

You Try!
The vertices of triangle XYZ are X(-4, 7), Y(0, 8) and Z(2, -1).

A) What are the vertices of $r_{(180,O)}$(triangle XYZ)?

B) What are the vertices of $r_{(270,\ O)}$ (triangle XYZ)?

State the number of turns and direction for each of the following rotations. Then state the coordinates of the image.



a) $r_{(90,\ O)}$ (C)

b) $r_{(180,\ O)}$ (G)

C) $r_{(-90,\ G)}$ (E)

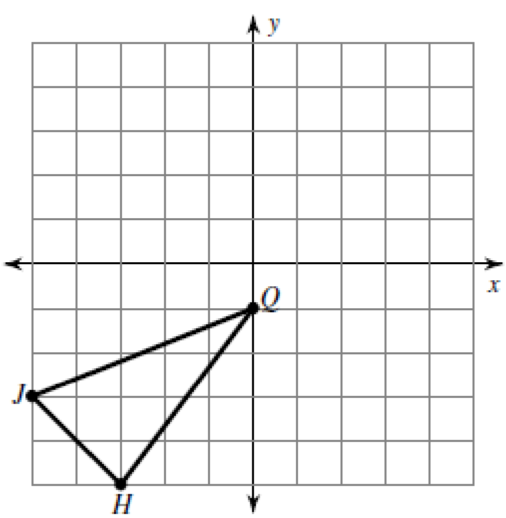
D) $r_{(90,\ H)}$ (D)

Practice:

1) Rotate ΔJQH -90° about the origin.

Which direction do you move?_______

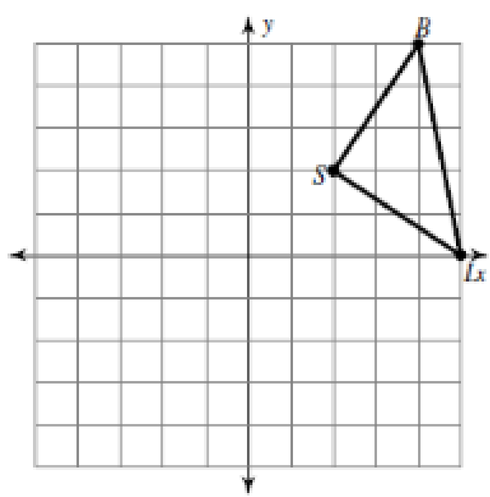
How many turns?_________



2) Rotate ΔSBL 180° about the origin.

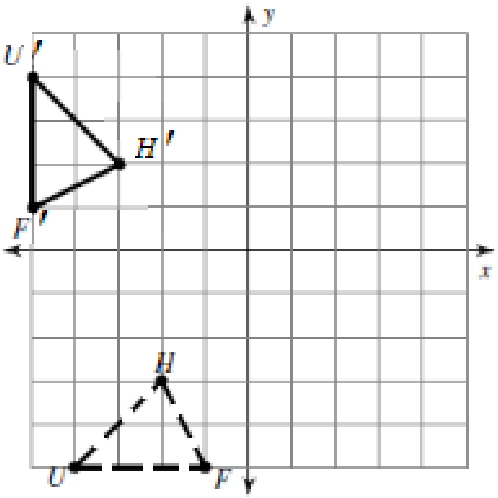
Which direction do you move?_______

How many turns?_________

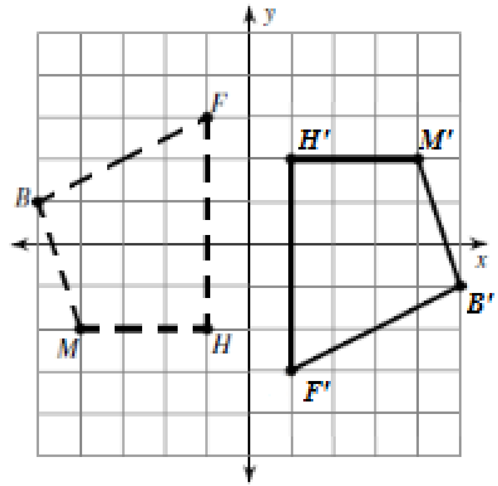


Describe the rotations below using one clockwise rotation and one counter-clockwise rotation.

3)



4)



This rotation could be described

as ______° CW, or ______° CCW

Write in Formal Notation:

This rotation could be described

as ______° CW, or ______° CCW

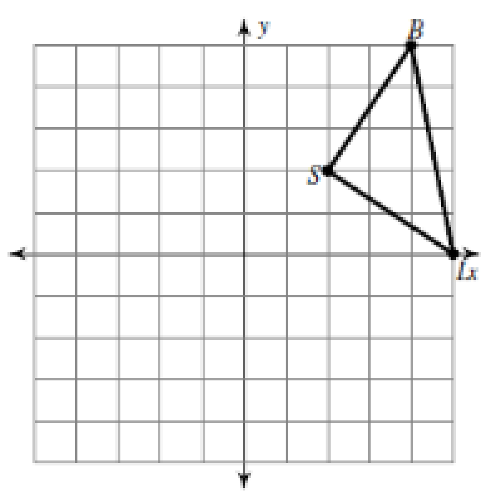
Write in Formal Notation:

5) Rotate the triangle below 270° about the point (1, 3).

Write in formal Notation:

Which direction do you move?_______

How many turns?_________



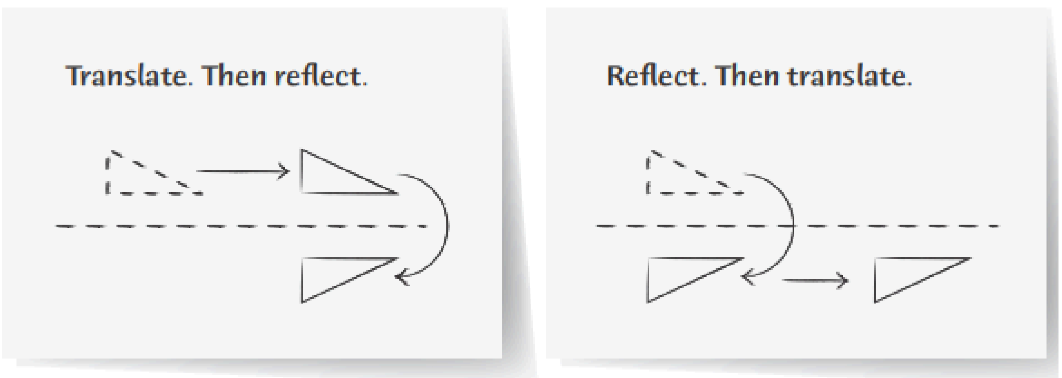
Day 4 - COMPOSITIONS (Lesson 8-4)

Looking at Compositions!

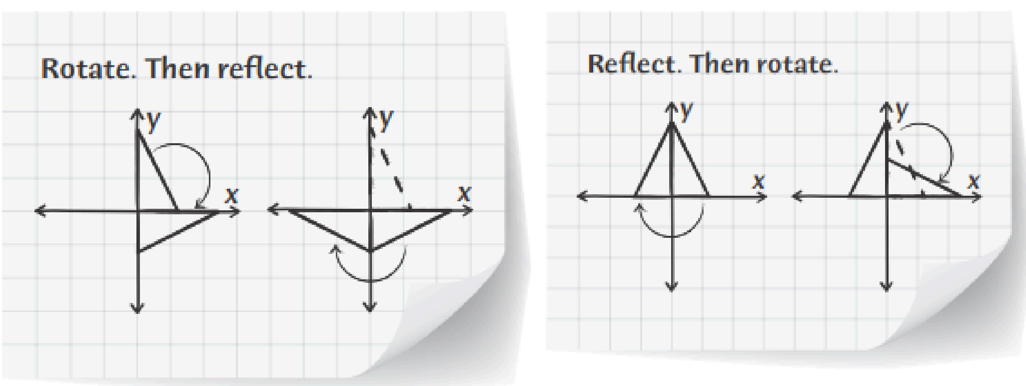
- A Composition is two or more ______________________________.

CRITIQUE & EXPLAIN

Two students are trying to determine whether compositions of rigid motions are commutative. Paula translates a triangle and then reflects it across a line. When she reflects and then translates, she gets the same image. She concludes that compositions of rigid motions are commutative.



Keenan rotates a triangle and then reflects it. When he changes the order of the rigid motions, he gets a different image. He concludes that compositions of rigid motions are not commutative.

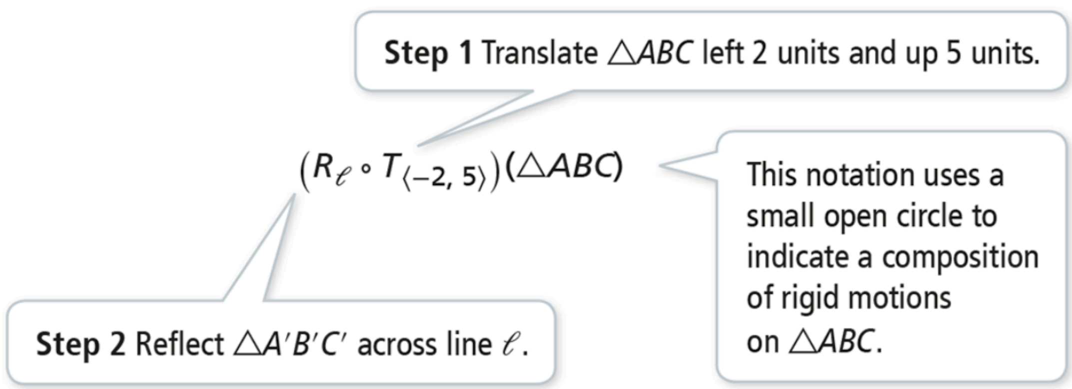


What does Commutative mean?

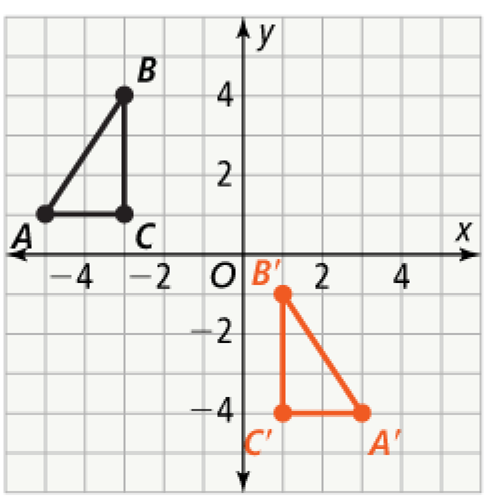
Do you agree with Paula or Keenan? Explain.

Should Paula have used grid paper? Explain.
What should you look for to determine whether two given rigid motions are commutative?

ORDER MATTERS!
Concept: A composition is a transformation with two or more rigid motions in which the second rigid motion is performed on the image of the first rigid motion.



Does a reflection, translation, or rotation map triangle ABC to triangle A’B’C’?

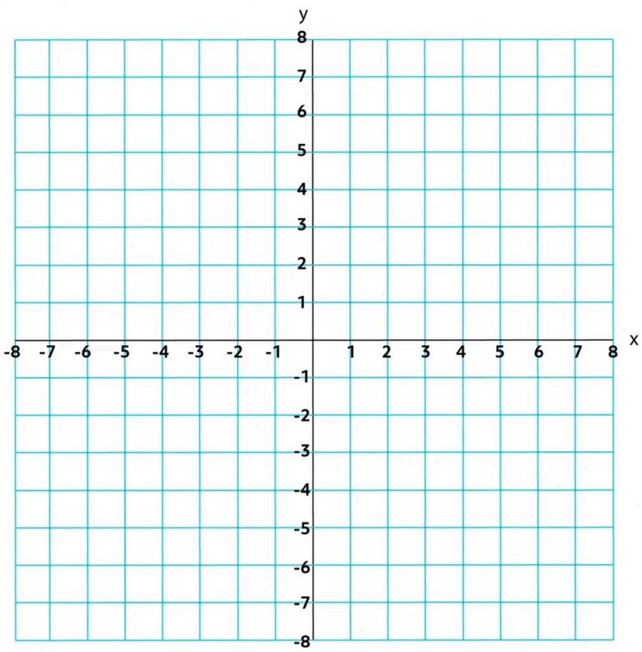


What composition of two rigid motions maps triangle ABC to triangle A’B’C’?

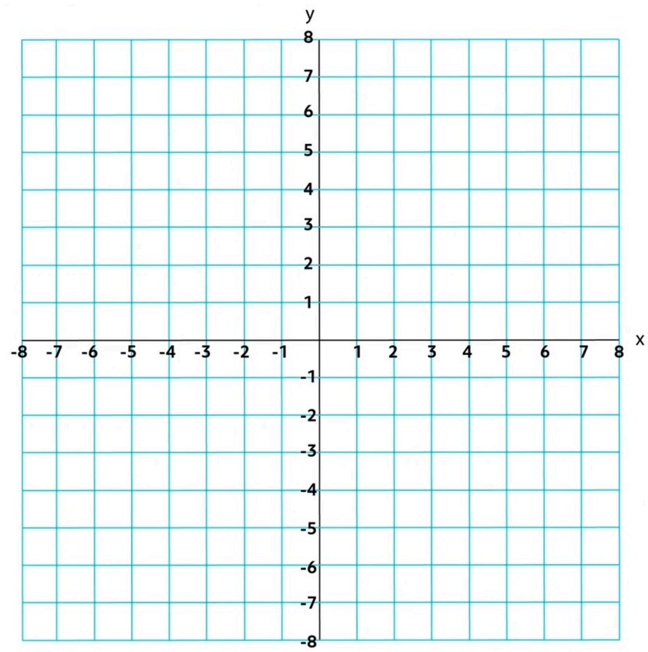
- The composition of a reflection followed by a translation in a direction parallel to the line of reflection is called a ________________ ____________________.

Complete the following Compositions:

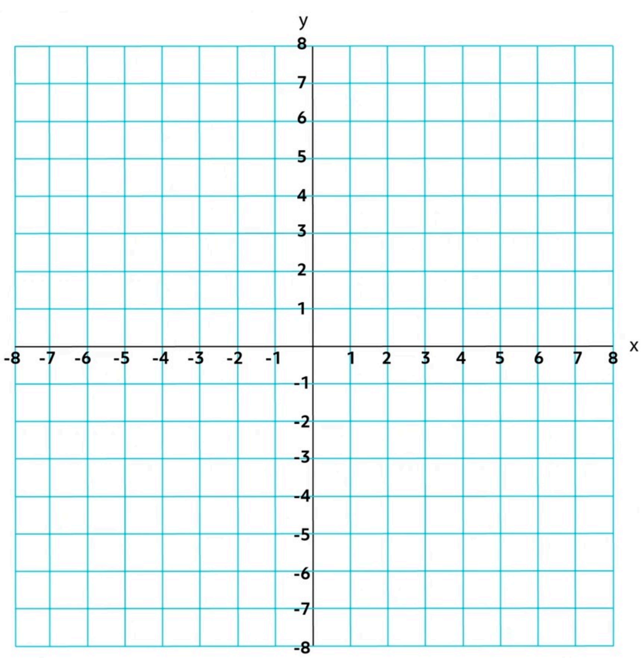
1. $(T_{(-4,3)} \circ R_{y=3})$



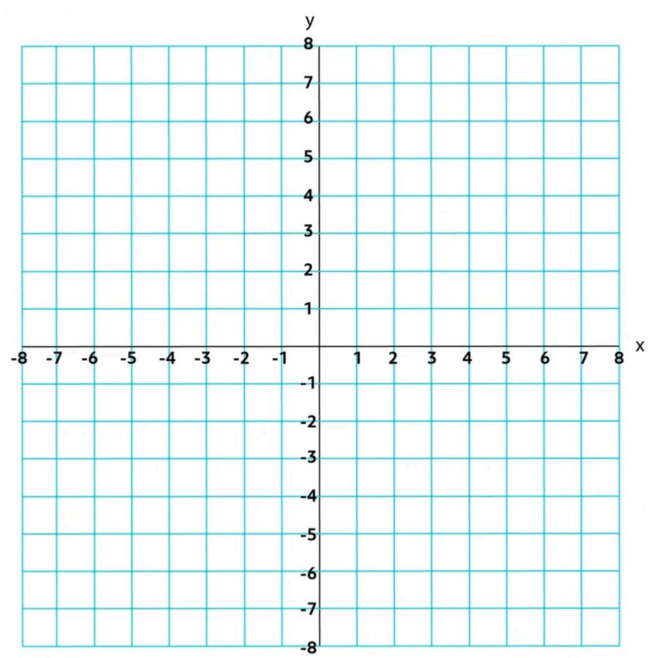
2. $(T_{(-4,0)} \circ R_{x-axis})$



3. $(T_{(3,5)} \circ R_{y-axis})$

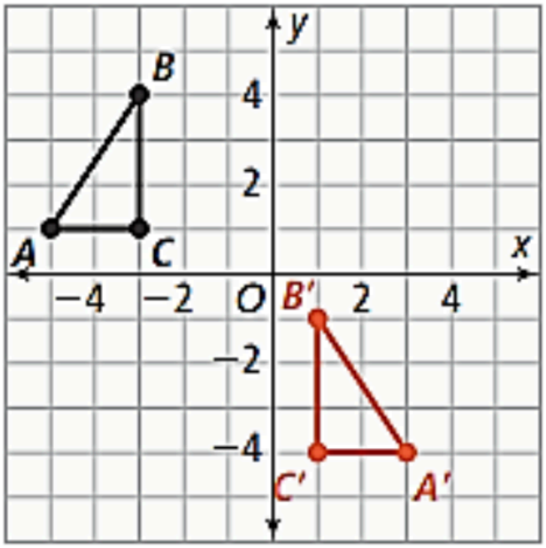


4. $(T_{(-4,-3)} \circ R_{x-axis})$

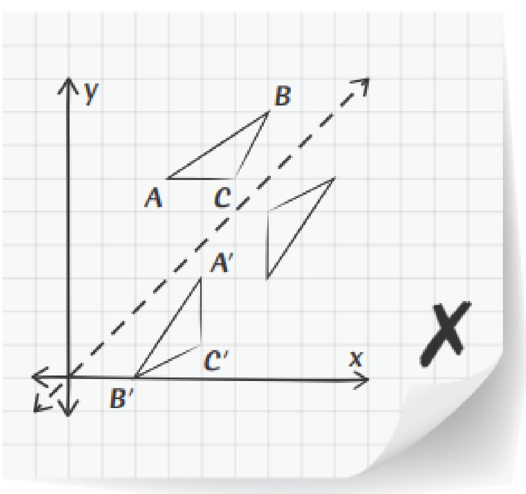


Practice:

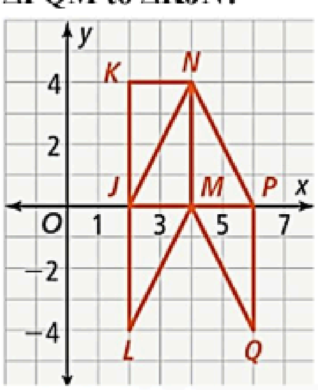
1.Explain why a reflection alone can or cannot map triangle ABC to triangle A'B'C'



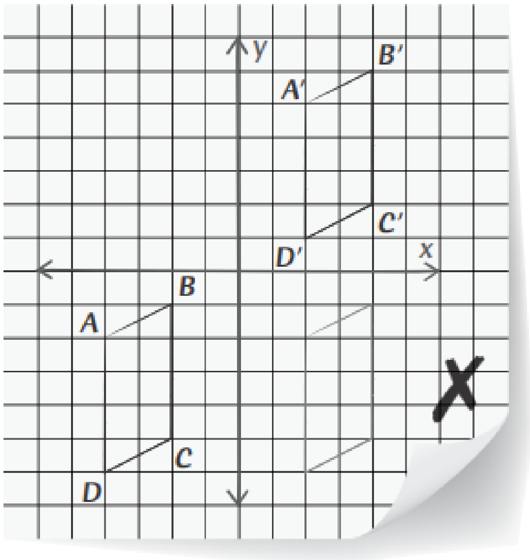
2.Tamika draws the following diagram as an example of a glide reflection. What error did she make?



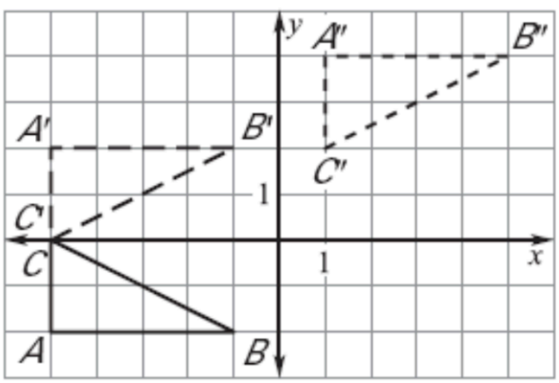
<u>ADDITIONAL EXAMPLE 3</u>: Does the glide reflection $R_n \circ T_{\langle 0,4 \rangle}$, where *n* is the line $x = 4$, map ΔPQM to ΔKJN?

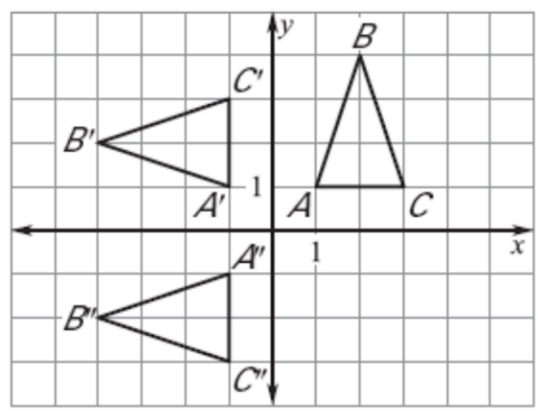


4. Damian draws the diagram for the glide reflection $(T_{(0,7)} \circ R_{y-axis})(ABCD)$. What error did he make?

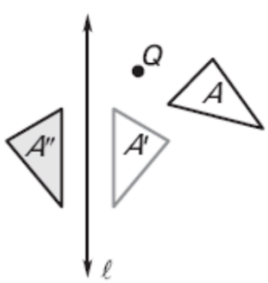


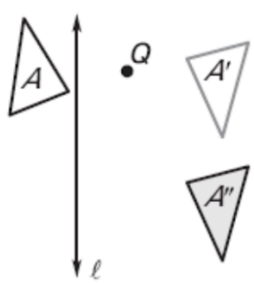
Describe the composition of the transformations.

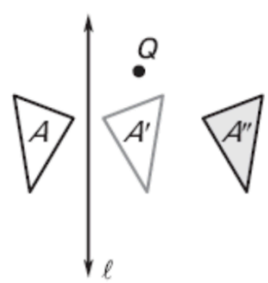
6) 

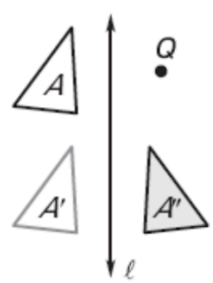
7) 

Match the composition with the diagram.

A. 

B. 

C. 

D. 

12) Translate parallel to ℓ then reflect in ℓ.

13) Rotate about Q, then translate parallel to ℓ.

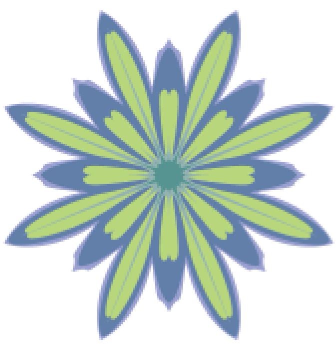
14) Rotate about Q, then reflect in ℓ.

15) Reflect in ℓ, then translate perpendicular to ℓ.

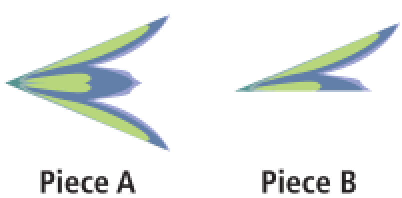
Day 5 - SYMMETRY _(Lesson 8-5)

EXPLORE & REASON

Look at the kaleidoscope image shown. Then consider pieces A and B taken from the image.

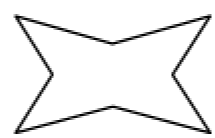


A) How are piece A and piece B related? Describe a rigid motion that you can use on piece B to produce piece A.

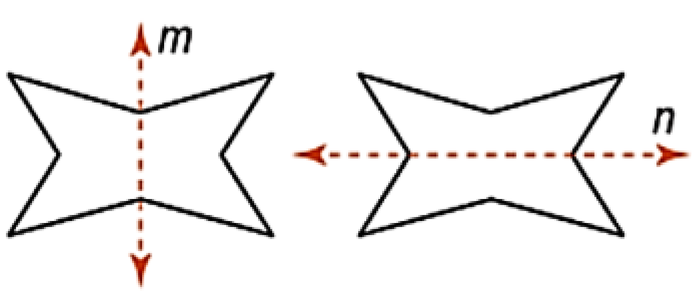


B) Describe a sequence of rigid motions that you can use on piece A to produce the image.

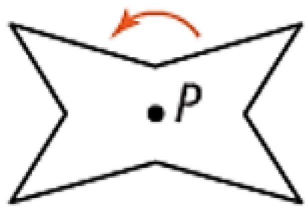
What transformations can be used to map the figure onto itself?



- A figure has ___________________ if a rigid motion can map the figure onto itself.
- __________________________ __________________________ is a symmetry for when a reflection maps the figure onto itself. The line of reflection for a reflectional symmetry is called the line of symmetry.



- A figure has ______________________________ ______________________________ if its image is mapped onto the preimage after a rotation of lesson than 360°.



- The type of symmetry for which there is a rotation of 180° that maps a figure onto itself is called ______________________________ ______________________________

Practice:

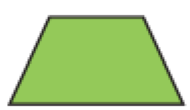
1. What transformations map the figures onto itself?

a. 

b. 

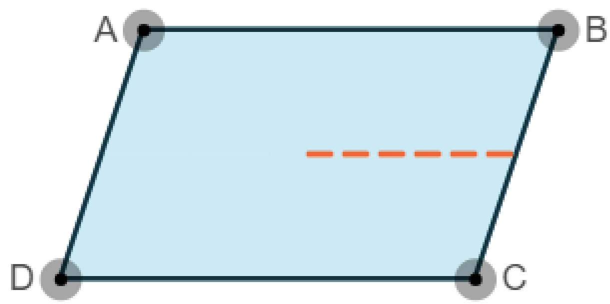
2. How many lines of symmetry does the figure have?

a. 

b. 

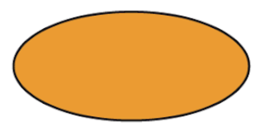
3. For what angles of rotation does the figure map onto itself?

A) A Parallelogram

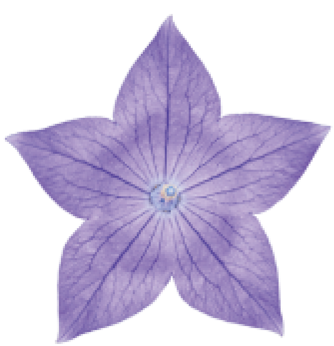


B) Irregular Shape

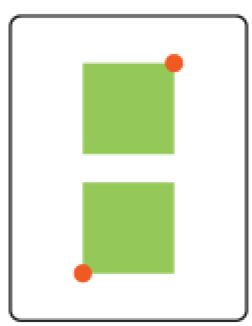
c) Oval



4. What types of symmetry does the figure have?



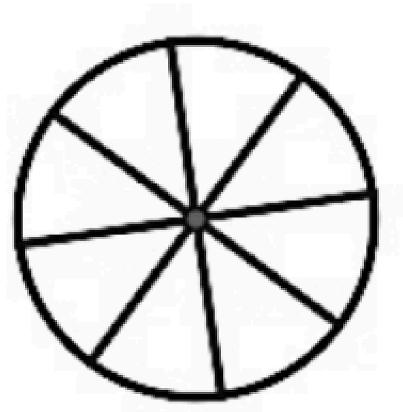
A)



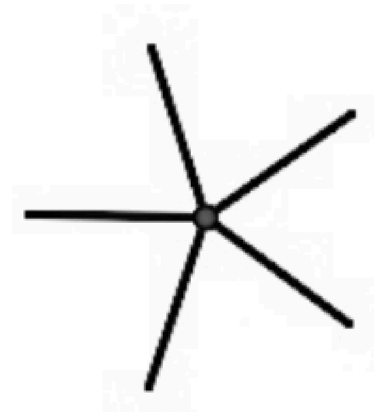
B)

Extra Practice!

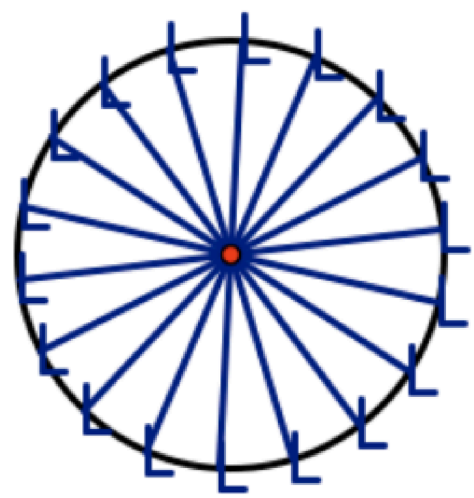
1. What fraction of a turn does the wagon wheel below need to turn in order to appear the very same as it does right now? How many degrees of rotation would that be?



2. What fraction of a turn does the propeller below need to turn in order to appear the very same as it does right now? How many degrees of rotation would that be?



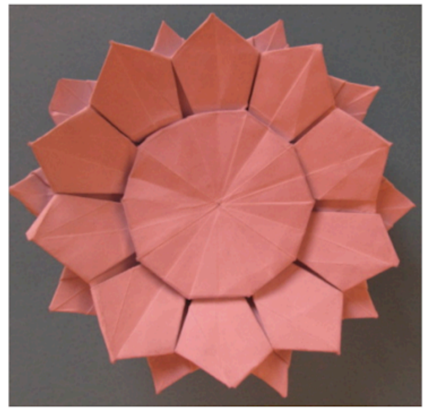
3. What fraction of a turn does the model of a Ferris wheel below need to turn in order to appear the very same as it does right now? How many degrees of rotation would that be?



4. How many sides does a regular polygon have that has an angle of rotation equal to 20?

Lesson 6 Symmetries of Regular Polygons

A Solidify Understanding Task

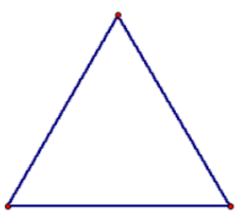


CC BY Jorge Jaramillo
https://flic.kr/p/rbd9zs

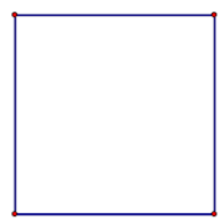
A line that reflects a figure onto itself is called a **line of symmetry**. A figure that can be carried onto itself by a rotation is said to have **rotational symmetry**. A **diagonal of a polygon** is any line segment that connects non-consecutive vertices of the polygon.

For each of the following regular polygons, describe the rotations and reflections that carry it onto itself: (be as specific as possible in your descriptions, such as specifying the angle of rotation)

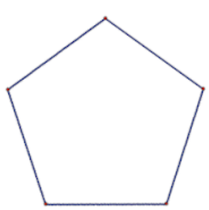
1. An equilateral triangle



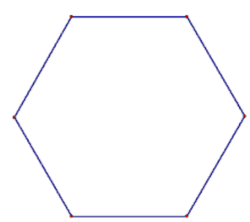
2. A square



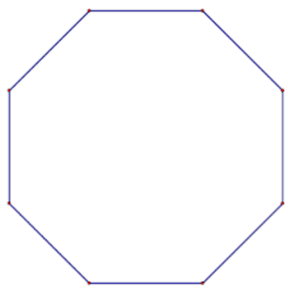
3. A regular pentagon



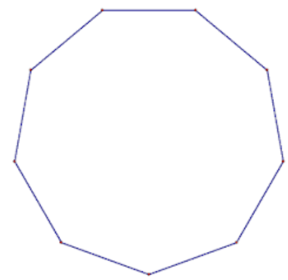
4. A regular hexagon



5. A regular octagon



6. A regular nonagon



What patterns do you notice in terms of the number and characteristics of the lines of symmetry in a regular polygon?

What patterns do you notice in terms of the angles of rotation when describing the rotational symmetry in a regular polygon?